Exercise sheet 2

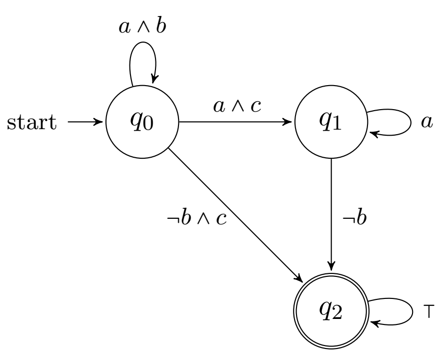
Submission deadline: Friday 20th December, 2019

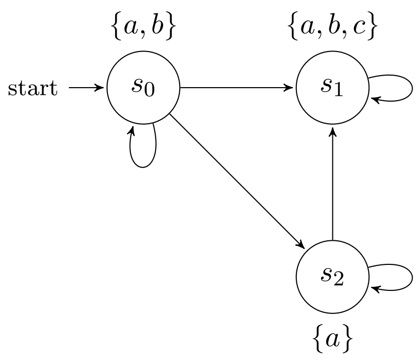
Submission: by e-mail to paul.wild@fau.de. Feel free to ask questions!

LTL model checking

1. Use the construction from the tutorials ([1], Theorem 5.37) to build a generalized Büchi automaton for the LTL formula $a \mathsf{U} (\mathsf{X} a \mathsf{U} a)$. (Hint: the final automaton should consist of five states.)

2. Consider the Büchi automaton $\mathcal{B}$ and the transition system $\mathcal{M}$ below:





We assume that the set of atomic propositions is $\mathcal{A} = \{a, b, c\}$. In the diagram for $\mathcal{B}$, an edge labelled by a propositional formula denotes a set of edges, one for each set $A \in \mathcal{P}\mathcal{A}$ satisfying that formula. For instance, the edge with label $\neg b \wedge c$ corresponds to two edges labelled with $\{c\}$ and $\{a, c\}$, respectively.

(a) Put $\varphi = (a \mathsf{U} \neg b) \wedge (b \mathsf{U} c)$. Show that $\mathcal{B}$ is an automaton for φ, that is its accepted language is exactly $\mathrm{Words}(\varphi) \subseteq (\mathcal{P}\mathcal{A})^\omega$ (see [1], Definition 5.6 for a precise definition).

(b) Build the product automaton $\mathcal{B} \otimes \mathcal{M}$. Instead of the definition in [1] (where states of $\mathcal{M}$ are paired with the *next* state in $\mathcal{B}$), use the following:

$$(s, q) \xrightarrow{A} (s', q') \text{ in } \mathcal{B} \otimes \mathcal{M} \quad :\Leftrightarrow \quad s \to s' \text{ in } \mathcal{M} \text{ and } q \xrightarrow{A} q' \text{ in } \mathcal{B} \text{ and } L(s) = A.$$

What should the sets of initial and final states be in this case?

(c) Use the automaton $\mathcal{B} \otimes \mathcal{M}$ to show that $s_0 \not\models \varphi$.

References

[1] C. Baier and J.-P. Katoen. *Principles of Model Checking (Representation and Mind Series)*. The MIT Press, 2008.