

Exercise sheet 4

Submission deadline: 2017-01-28

Submission: by e-mail to christoph.rauch@fau.de. Feel free to ask questions!

Some XPath Equivalences (6 Points)

1. Recall the basic axiom schemes for Core XPath, i.e. ISAx1–7, PrAx1–4, and NdAx1–4. From the latter two groups of axioms, choose two axioms each and show that they are valid in the XML tree semantics for Core XPath.
2. From these axioms, further useful equivalences can be derived using Birkhoff's Calculus for equational logic. Show the following in that way:

$$(A \cup B)[\phi] \equiv A[\phi] \cup B[\phi] \quad (1)$$

Simple Node Expressions (8 Points)

In the lecture, we gave translation functions from node expressions in Core XPath to simple node expressions in the grammar

$$siNode(A) := true \mid p \mid \langle a[siNode] \rangle \mid \neg siNode \mid siNode \vee siNode$$

as follows:

$$\begin{array}{ll} p^s := p & .^{s'}(\phi) := \phi \\ (\neg\phi)^s := \neg(\phi^s) & a^{s'}(\phi) := \langle a[\phi] \rangle \\ (\phi \vee \psi)^s := \phi^s \vee \psi^s & (A \cup B)^{s'}(\phi) := A^{s'}(\phi) \vee B^{s'}(\phi) \\ \langle A \rangle^s := A^{s'}(true) & (A[\psi])^{s'}(\phi) := A^{s'}(\psi^s \wedge \phi) \\ & (A/B)^{s'}(\phi) := A^{s'}(B^{s'}(\phi)) \end{array}$$

Prove the following lemma:

Lemma. For every path expression A and for every node expression ϕ ,

$$\langle A[\phi] \rangle \equiv A^{s'}(\phi^s) \quad \text{and} \quad \phi \equiv \phi^s \quad (2)$$

are provable.

In addition to (1) and the axioms, you might need the derived equivalence

$$A[\phi][\psi] \equiv A[\phi \wedge \psi] \quad (3)$$

(you do **not** need to prove it). **Caution:** be careful about the distinction between regular node expressions and simple node expressions at all times.