Logical Foundations of Databases WS 2016/17 November 10, 2016

Exercise sheet 1: First-Order Logic

Submission deadline: 2016-11-17

Submission: by e-mail to christoph.rauch@fau.de. Feel free to ask questions!

1 Deduction using Hilbert System (4 Points)

Derive the following tautologies using the Hilbert-style deduction system for local derivability described e.g. on slide 5 of lecture 3:

- $\vdash_{\mathbf{loc}} \alpha \to ((\alpha \to \bot) \to \bot)$
- $\vdash_{\mathbf{loc}} (\alpha \to (\beta \to \bot)) \to (\alpha \to (\beta \to \gamma))$

Hint: You are allowed to use $p \implies p$ like an axiom, since we proved it during the tutorial

2 Sequent Calculus (6 Points)

Prove the following judgements using the sequent calculus seen in the lecture:

1. $\forall v\,.(\alpha \to \beta) \vdash_{\mathbf{loc}} (\exists v.\,\alpha) \to \beta$, where v is fresh for β
2. $\Gamma_{deg} \vdash_{\mathbf{loc}} \forall xy.\, x \approx y$, where

$$\Gamma_{deg} := \{\forall xyz.\, Flight(x, y) \to Flight(x, z) \to y \approx z, \forall xy.\, Flight(x, y) \vee Flight(y, x)\}$$

Hint: You may find a tool like `http://logitext.mit.edu` helpful in carrying out the proofs. E.g. the second judgement can be entered there as follows:

```
(forall x y z, F(x,y) -> F(x,z) -> E(y,z)),
  (forall x y, F(x,y) \/ F(y,x)) |- (forall x y, E(x,y))
```

where `E(x,y)` is used to denote $x \approx y$. Note, however, that the proof tree can become quite large, so for your submission, try to find suitable abbreviations for formulas in the context and maybe derive additional rules as done in the tutorial. It can also be helpful to split the proof into multiple trees.