Sheet 1

Due 03.11.2015

Exercise 1   Derivable Rules   (5 Points)

Derive left and right logical rules for the derived connectives $\vee$, $\rightarrow$.

Exercise 2   Some Sequent Proofs   (5 Points)

Derive the following formulas in the two-sided calculus, using also the rules established in Exercise 1:

1. $A \vee \neg A$

2. $(A \wedge B \rightarrow C) \rightarrow (A \rightarrow B \rightarrow C)$

3. $(A \rightarrow C) \rightarrow ((B \rightarrow C) \rightarrow ((A \vee B) \rightarrow C))$

Exercise 3   No Sequent Proof   (5 Points)

Conduct an exhaustive (hopefully, failing) proof search on the formula

$$\neg((\neg(A \wedge \neg B) \wedge \neg(B \wedge \neg C)) \wedge \neg(C \wedge \neg A)),$$

using only the rules for $\wedge$ and $\neg$. Which satisfying valuations for the negation of the formula can be read off the branches of the proof search?

Exercise 4   Structural Rules   (5 Points)

Show by induction on proofs that the left and right contraction rules

$$(LC) \ \frac{\Gamma, A, A \vdash \Delta}{\Gamma, A \vdash \Delta} \qquad (RC) \ \frac{\Gamma \vdash \Delta, A, A}{\Gamma \vdash \Delta, A}$$

are admissible.