NP Reasoning in the Monotone μ-Calculus

(joint work with Daniel H.)

K / ALC Sat. checking PSPACE

K + global axioms / universal mod. EXPTIME

μ-Calculus EXPTIME

Monotone Modal Logic

$$\varphi, \psi ::= \bot \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid p$$

neighbourhood frames / models

$M = (W, N, I)$ $\quad I : At \to \mathcal{P}(W)$

$N : W \to \mathcal{P}(\mathcal{P} W)$

$$w \models \Box\varphi \iff \forall S \in N(w)\ \exists w' \in S.\ w' \models \varphi$$

$$w \models \Diamond\varphi \iff \exists S \in N(w)\ \underbrace{\forall w' \in S.\ w' \models \varphi}_{S \subseteq [\![\varphi]\!]}$$

$$K: \frac{\Gamma, \Box\varphi_1, \dots, \Box\varphi_m, \neg\Box\varphi_0}{\underbrace{\varphi_1, \dots, \varphi_m, \neg\varphi_0}_{O(n)}} \qquad M: \frac{\Gamma, \Box\varphi, \neg\Box\psi}{\underbrace{\varphi, \neg\psi}_{O(1)}}$$

Vardi: 1989: M is in NP

easy: same under global assumptions

(W, N, I) φ-model $\Leftrightarrow$ $[\![\varphi]\!] = W$

ψ φ-satisfiable $\Leftrightarrow$ ψ satisfiable in a φ-model

global assumption

universal modality: $\ldots \mid \underline{A}\varphi$

$w \models \underline{A}\varphi \Leftrightarrow \forall w' \in W.\ w' \models \varphi$

guess whether $\underline{A}\varphi$ is true, replace with $\top / \bot$

$\rightarrow$ Global ass'n $\bar{\varphi}$,

for $\underline{A}\varphi$ false check $\neg\varphi$ $\bar{\varphi}$-sat.

μ-calculus:

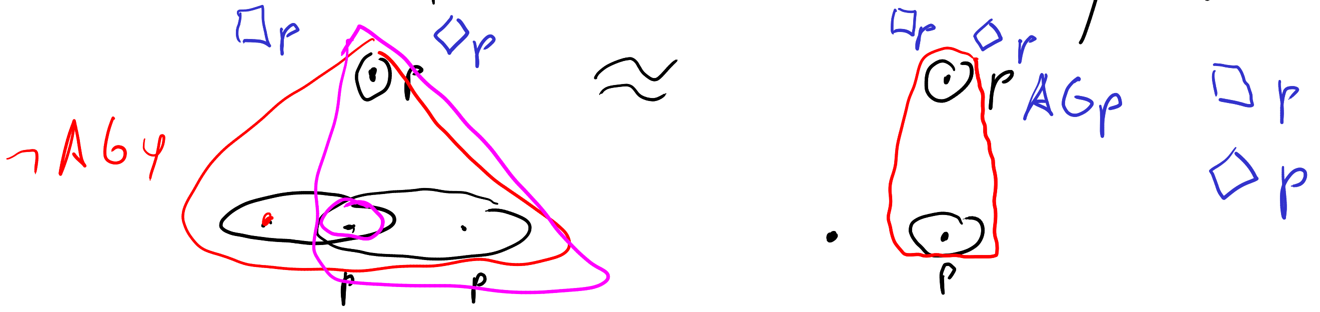
$$\varphi, \psi ::= \bot \mid \top \mid \varphi \wedge \psi \mid \varphi \vee \psi \mid p \mid \neg p \mid \mu X.\varphi \mid \nu X.\varphi \mid X \mid \Box\varphi \mid \Diamond\varphi$$

relationally: $AG\varphi := \nu X.\ \varphi \wedge \Box X$

$w \models \underbrace{AG}_{\text{submodel modality}} \varphi \iff \exists$ closed submodel M'. $w \in W'$, M' φ-Model

M' submodel of $M \iff M' \hookrightarrow M$ bisimulation

AG not expressible in monotone μ-calculus:



- $\langle a \rangle$ „Agent a knows"
- CPDL (Peleg 1987)

$\triangleq$ PDL $[\alpha]\varphi$

$\alpha ::= ?\varphi \mid \alpha_1 ; \alpha_2 \mid \alpha_1 \cup \alpha_2 \mid a \mid \alpha^* \mid \alpha_1 \cap \alpha_2$

$\langle \alpha_1 \cup \alpha_2 \rangle \varphi \equiv \langle \alpha_1 \rangle \varphi \wedge \langle \alpha_2 \rangle \varphi$

$\langle \alpha_1 \cap \alpha_2 \rangle \varphi \equiv \langle \alpha_1 \rangle \varphi \vee \langle \alpha_2 \rangle \varphi$

$S \in N(\alpha, w)$

- Game logic (Parikh 1985)

$\ldots \mid \alpha^d$ $\qquad \alpha^\times = ((\alpha^d)^*)^d$

$(a^*+b)^*$

$\langle a^* \rangle \varphi = \nu X. \varphi \wedge \langle a \rangle X$ guarded (?)

CPDL $\subseteq$ Game logic (alternation-free) $\subseteq$ alternation-free monotone μ-calculus

Peleg: CPDL is EXPTIME-complete

Hamm.. / LS: alternation-free monotone μ-calculus + $\underline{A}$ is in NP

atomic programs as relations $\Rightarrow$ PDL $\subseteq$ CPDL
(sequential)