Permutation Games for the Weakly Aconjunctive μ-Calculus

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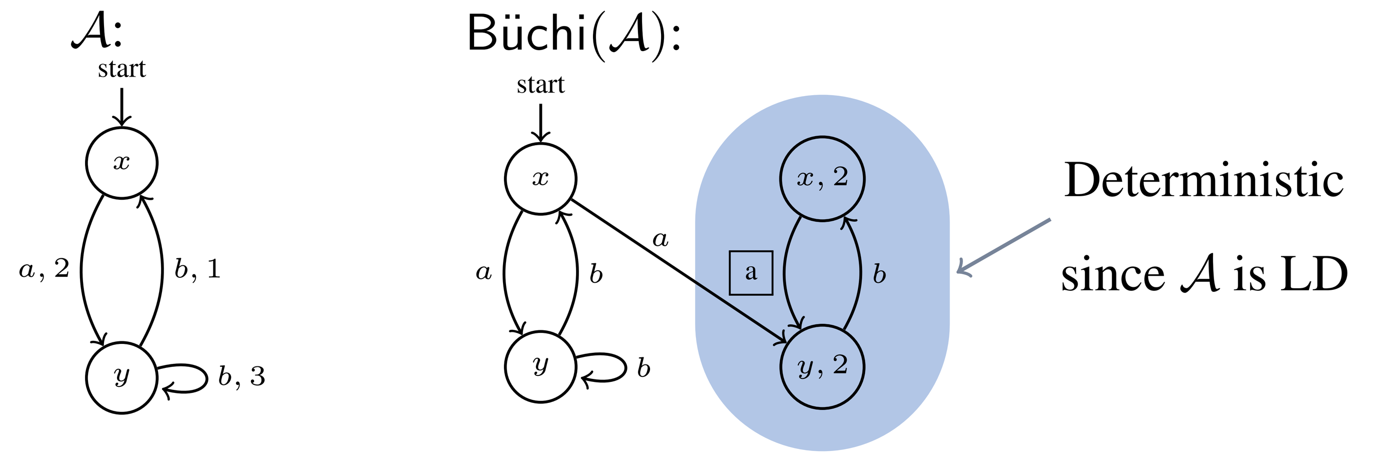
From Limit-deterministic Parity Automata to Deterministic Parity Automata

A parity automaton (PA) or Büchi automaton (BA) is *limit-deterministic* (LD) if every accepting run in it is deterministic from some point on.

Limit-deterministic PA to Limit-deterministic BA

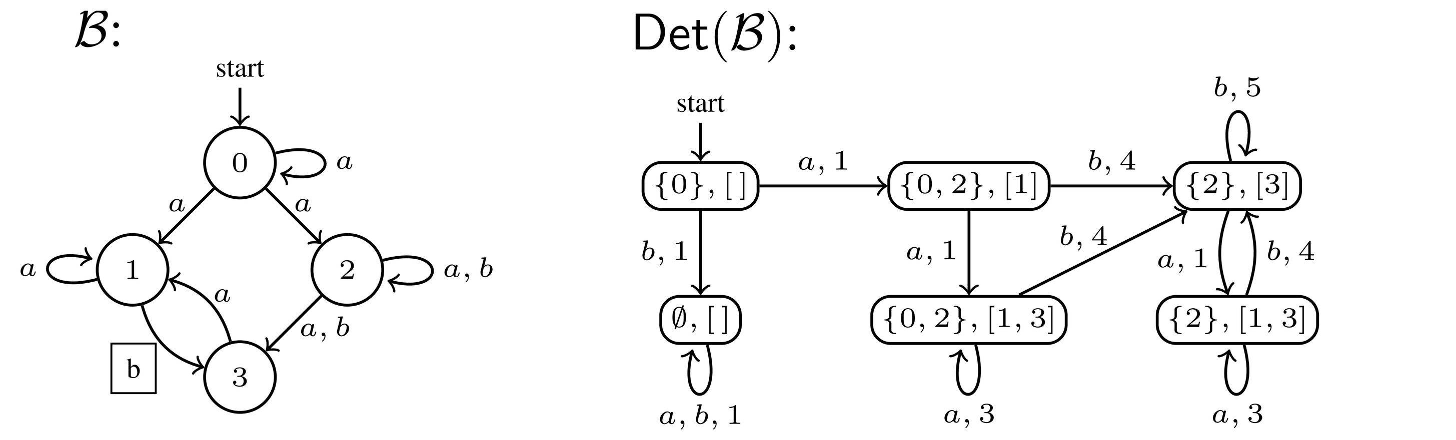
Eventually guess even highest priority. E.g.:



Transforms LDPA $\mathcal{A}$ of size n with k priorities to LDBA Büchi($\mathcal{A}$) of size $\mathcal{O}(nk)$

Limit-deterministic BA to DPA

Safraless determinization, using *permutations* of states. E.g.:



Determinizes LDBA $\mathcal{B}$ of size n to DPA Det($\mathcal{B}$) of size $\mathcal{O}(n!)$ with $\mathcal{O}(n)$ priorities

Theorem: If $\mathcal{A}$ is a limit-deterministic PA of size n with k priorities, then $L(\mathcal{A}) = L(\mathsf{Det}(\mathsf{Büchi}(\mathcal{A})))$; the latter is of size $\mathcal{O}((nk)!)$ and has $\mathcal{O}(nk)$ priorities.

The Aconjunctive μ-Calculus, Limit-deterministic Tracking Automata, Permutation Games

The Aconjunctive μ-Caculus [Kozen, 1983]

Modal formulas with fixpoint variables and fixpoint operators; e.g. $\nu X.\, \mu Y.\, ((p \wedge \Box X) \vee (\neg p \wedge \Box Y))$ ("on all paths, infinitely often p"), interpreted over standard Kripke structures. Formula ϕ is *aconjunctive* if for all conjunctions occurring as subformula in ϕ, at most one conjunct contains an *active* μ-variable. ("All fixpoints have at most one trace per path")

Construct *tracking automaton* $\mathcal{A}_\phi$ for ϕ, i.e. a parity automaton that tracks subformulas of ϕ through rule applications in pre-tableaux.

Example (aconjunctive formula ϕ, tracking automaton $\mathcal{A}_\phi$ and pre-tableau for ϕ)

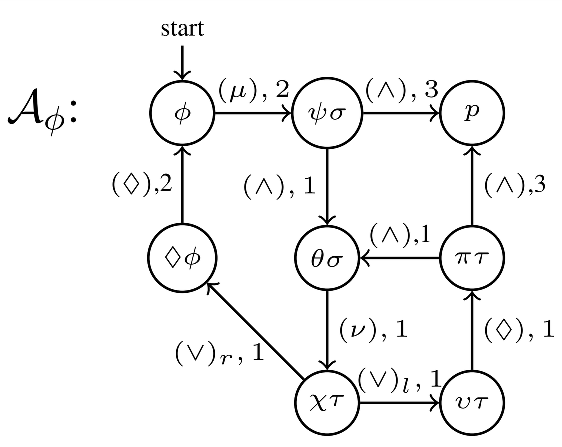
$$\phi = \mu X.\, (p \wedge \nu Y.\, (\Diamond(Y \wedge p) \vee \Diamond X))$$

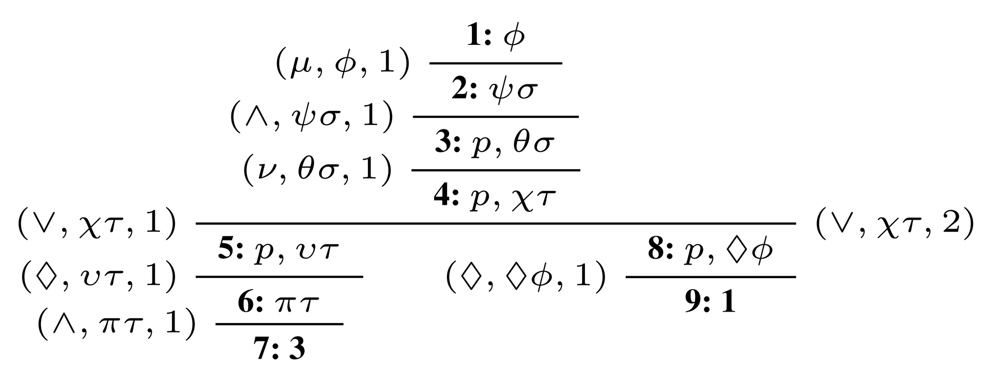
$\phi := \mu X.\psi$ $\qquad$ $\psi := p \wedge \theta$

$\theta := \nu Y.\, \chi$ $\qquad$ $\chi := \upsilon \vee \Diamond X$

$\upsilon := \Diamond \pi$ $\qquad$ $\pi := Y \wedge p$

$\sigma := [X \mapsto \phi]$ $\qquad$ $\tau := [Y \mapsto \theta]; \sigma$





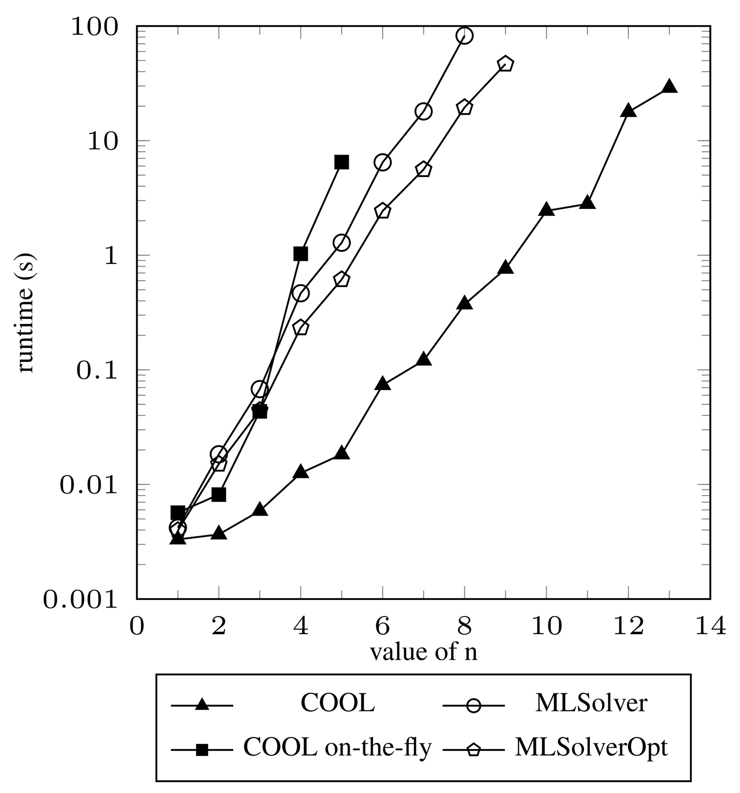
Theorem: If ϕ is aconjunctive, then the tracking automaton $\mathcal{A}_\phi$ is a limit-deterministic PA of size $\mathcal{O}(|\phi|)$ and with $\mathcal{O}(\mathsf{ad}(\phi))$ priorities.

Then determinize $\mathcal{A}_\phi$ through limit-deterministic BA with permutation method; complement resulting DPA; obtain DPA $\mathcal{D}_\phi$ that accepts exactly the *good branches* (in which no least fixpoint is unfolded infinitely often) in pre-tableaux for ϕ. Build *permutation game* $\mathcal{G}_\phi$ over carrier of $\mathcal{D}_\phi$.

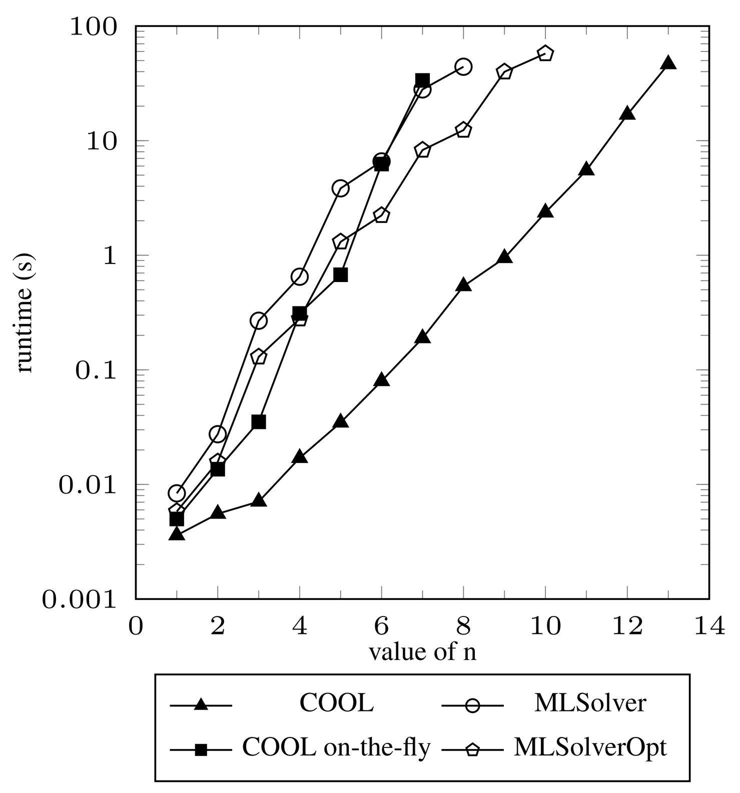
Theorem: For aconjunctive ϕ, Eloise wins $\mathcal{G}_\phi$ if and only if ϕ is satisfiable in a model of size $\mathcal{O}((nk)!)$, where $n = |\phi|$ and $k = \mathsf{ad}(\phi)$.

Implementation as part of COOL (Coalgebraic Ontology Logic Reasoner)

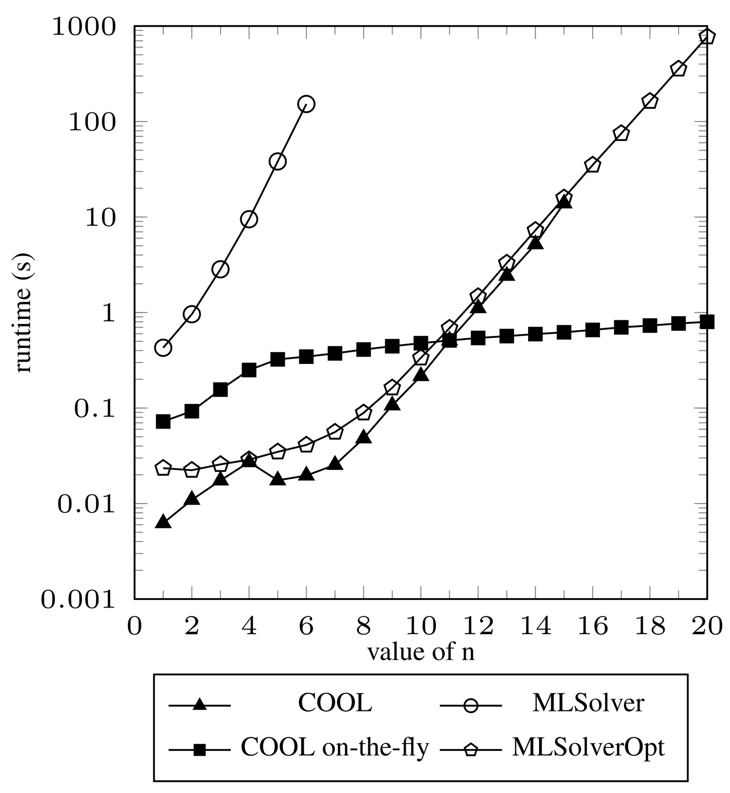
Solves permutation satisfiability games *on-the-fly* and in *coalgebraic* generality; `https://www8.cs.fau.de/research:software:cool`



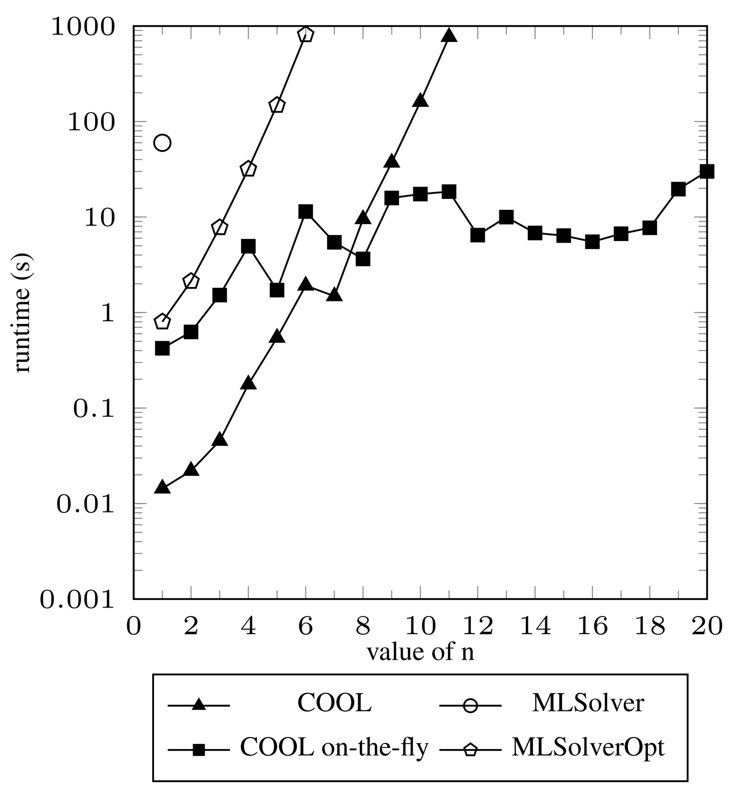
$\phi_{\mathsf{aut}}(n) \to (\phi_{\mathsf{ne}}(n) \leftrightarrow \bigvee_{i \text{ even}} \mu X.\nu Y.\mu Z.\ \theta_{\Diamond}(i))$



$\phi_{\mathsf{game}}(n) \to (\phi_{\mathsf{win}}(n) \to \bigwedge_{i \text{ odd}} \nu X.\mu Y.\nu Z.\ \phi_{\mathsf{strat}}(\theta_{\heartsuit}(i)))$



early-ac$(n, 4, 2)$



early-ac$_{gc}(n, 4, 2)$

where $\theta_{\heartsuit}(i) = (q_i \wedge \heartsuit Y) \vee \bigvee_{i<j\leq n}(q_j \wedge \heartsuit X) \vee \bigvee_{1\leq j\leq i}(q_j \wedge \heartsuit Z)$ and $\phi_{\mathsf{strat}}(\psi_{\heartsuit}) = (q_e \wedge \psi_{\Diamond}) \vee (q_a \wedge \psi_{\Box})$