Description Logics and Formal Ontologies Winter semester 2025/26

Exercise Sheet 1

Due: 2025-11-03

Please include the names of all group members on your hand-in.

Exercise 1 (5 Points)

Apply the tableaux algorithm from the lecture to the formula

$$((p \wedge q) \rightarrow \neg r) \wedge (\neg p \rightarrow r) \wedge (\neg q \rightarrow r) \wedge \neg(q \rightarrow \neg r).$$

Is the formula satisfiable? [Hint: First encode all connectives in terms of $\neg$ and $\wedge$.]

Exercise 2 (5 Points)

Apply the resolution algorithm to the CNF

$$\{\neg p, q, r\}, \{\neg q, u\}, \{\neg r, u\}, \{\neg u, \neg p, \neg q\}, \{\neg r, \neg p\}, \{q, p\}, \{\neg u, p\}.$$

Is the CNF satisfiable? Subsequently, apply DPLL instead – do simplifications arise? Which optimizations apply?

Exercise 3 Non-atomic Axiom Rule (5 Points)

Show that instead of the axiom rule $\Gamma, p, \neg p / \bot$ of the propositional tableau calculus given in the leccture, one may equivalently use the stronger rule

$$\frac{\Gamma, \phi, \neg\phi}{\bot}$$

where ϕ now ranges over unrestricted formulae instead of just atoms. That is, show that even in the original calculus, $\Gamma, \phi, \neg\phi$ is unsuccessful. Use induction on ϕ.

Exercise 4 Positive Logic (5 Points)

A CNF is *positive* if all its clauses consist of positive literals only. Show that a positive CNF ψ is a logical consequence of a positive CNF ϕ ($\phi \models \psi$) if and only if every clause of ψ contains one of the clauses of ϕ. Argue semantically. Formulate the dual statement (which then characterizes logical consequence on positive DNFs).