Alternating Nominal Automata with Name Allocation

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Lehrstuhl für Theoretische Informatik 8
Friedrich-Alexander-Universität Erlangen-Nürnberg

Friedrich-Alexander-Universität
Faculty of Engineering

Motivation

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Motivation

- Consider sequences of user logins within a given time period on a server.

  Forming finite *data* words $a_1 \cdots a_n \in \mathbb{A}^\star$

$\mathbb{A}$: admissible user IDs for a server ($\rightsquigarrow$ *infinite set*)

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$$L_1 = \{a_1 \cdots a_n \in \mathbb{A}^\star \,:\, a_i = a_j \text{ for some } i \neq j\}$$

'some user has logged in twice'

'first pair of users is equal to last pair with only different users in between'

$$L_2 = \left\{a_1 \cdots a_n \in \mathbb{A}^\star \,:\, \left(\begin{array}{c} a_1 = a_{n-1} \wedge a_2 = a_n \wedge \\ \forall 2 \leqslant i < n-1.\, a_1 \neq a_i \wedge a_2 \neq a_{i+1} \end{array}\right)\right\}$$

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- **Now:** Model these patterns with explicit 'name binding' (*bar languages*)

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T.CS

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- Examples:

$$\underbrace{|a}_{} \; b \; a \; \underbrace{|b}_{} \; a \; b \; \underbrace{|a}_{} \; b \; a$$

'variable' intros

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$${|}a \;\; b \;\; a \;\; {|}b \;\; a \;\; b \;\; {|}a \;\; b \;\; a$$

$$n \;\; \boxed{b} \;\; n \;\; d \;\; n \;\; d \;\; a \;\; d \;\; a$$

a	n

$$h \;\; \boxed{b} \;\; h \;\; a \;\; h \;\; a \;\; a \;\; a \;\; a$$

a	h

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$$\cancel{n}\ \cancel{b}\ \cancel{n}\ \cancel{a}\ \cancel{n}\ \cancel{a}\ \cancel{a}\ \cancel{a}\ \cancel{a}$$

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a	

b	d
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~~$n\ b\ n\ a\ n\ a\ a\ a\ a$~~

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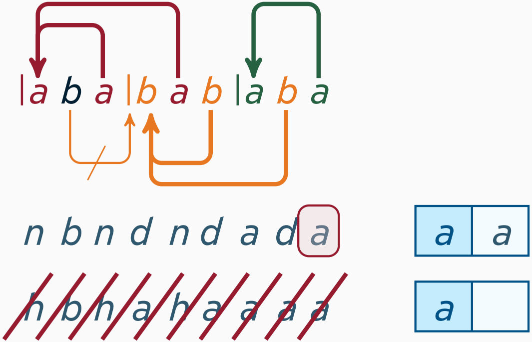

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$$|a\; b\; a\; |b\; a\; b\; |a\; b\; a \quad \equiv_\alpha \quad |c\; b\; c\; |d\; c\; d\; |e\; d\; e$$

» These 'bar strings', i.e. words with 'variables', result in different kinds of languages:

» *Data Languages* (over $\mathbb{A}^\star$)

» *Literal Languages* (over $\overline{\mathbb{A}}^\star$)

» *Bar Languages* (over $\overline{\mathbb{A}}^\star / \equiv_\alpha$)

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$\approx$

$\rightsquigarrow$ Fix a (countably infinite) set $\mathbb{A}$ of 'names'. ← Data Values

Definition (*Nominal Sets*) Gabbay, Pitts '99

A *nominal set* is a set whose elements depend on a *finite* number of these names. $\rightarrow \mathsf{supp}(x)$

$\rightsquigarrow$ We can change the names of an element using permutations $\pi\colon \mathbb{A} \xrightarrow{\simeq} \mathbb{A}$ which act upon these elements. ($\rightsquigarrow$ Group Actions $\triangleright\colon \mathsf{Perm}(\mathbb{A}) \times X \to X$)

```
<book id="bk007">
   <author lstname="Doe"
       fstname="John"/>
   <title value="Biggy"/>
   <price cur="USD"
       amount="12.95"/>
</book>
```

Nominal Sets (Talking about Infinite Data with Finite Means)

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```

Example: Let $\pi = (\text{John} \leftrightarrows \text{Jane}) \circ (\text{Doe} \leftrightarrows \text{Biggy})$ act on the document.

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TCS

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⋙ 'Freshness': $\mathbb{A} \ni a\#x$ iff $a \notin \mathsf{supp}(x)$.

⋙ Proper 'finiteness' is now replaced by finiteness up to such permutations. ⇝ *Orbit-Finiteness*

⋙ Nominal Sets and action-preserving maps form a category $\mathbf{Nom}$.

$$\forall \pi.\ \forall x.\ f(\pi \triangleright x) = \pi \triangleright f(x)$$

Definition (*Abstraction*) Gabbay, Pitts '99

Given a nominal set X, define the equivalence relation $\approx_\alpha$ on $\mathbb{A} \times X$ as follows:

$$(a, x) \approx_\alpha (b, y) \text{ iff } \exists c \# (a, b, x, y).\ (a\ c) \triangleright x = (b\ c) \triangleright y. \tag{1}$$

With this, define the nominal set $[\mathbb{A}]X$ as the quotient $(\mathbb{A} \times X)/\approx_\alpha$, and denote the equivalence classes by $\langle a \rangle x$.

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  Substitution is pushed back to the definition of the nominal set.

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Definition ($\equiv_\alpha$ *on Bar Strings* **)** **Schröder, Kozen, Milius, Wißmann '17**

$\equiv_\alpha$ is the equivalence generated by $w|av \equiv_\alpha w|bu$ if $\langle a \rangle v = \langle b \rangle u$ holds in $[\mathbb{A}]\overline{\mathbb{A}}^\star$

Bar Languages (With corresponding Data Languages)

$$\overline{\mathbb{A}} \;:=\; \mathbb{A} \;\cup\; \{|d \,:\, d \in \mathbb{A}\} \;\cong\; \mathbb{A} + \mathbb{A}$$

- A name a is *free* in a bar string $w \in \overline{\mathbb{A}}^{\star}$, if a occurs to the left of any occurence of $|a$.

$$\overline{\mathbb{A}} := \mathbb{A} \cup \{|d : d \in \mathbb{A}\} \cong \mathbb{A} + \mathbb{A}$$

- A name a is *free* in a bar string $w \in \overline{\mathbb{A}}^\star$, if a occurs to the left of any occurence of $|a$.
- A bar string $w \in \overline{\mathbb{A}}^\star$ is *clean* if its bound letters $|a$ are mutually distinct and distinct from all its free names.

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$$\overline{\mathbb{A}} \;:=\; \mathbb{A} \;\cup\; \{|d \,:\, d \in \mathbb{A}\} \quad\cong\quad \mathbb{A} + \mathbb{A}$$

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$$\mathsf{LF}(L_1) = \{a_1 \cdots a_n \in \mathbb{A}^\star \ : \ a_i = a_j \text{ for some } i \neq j\}$$

$$L_1 = [(|b)^*|a(|b)^* a(|b)^*]_\alpha$$

'some user has logged in twice'

'first pair of users is equal to last pair with only different users in between'

$$\mathsf{LF}(L_2) = \left\{a_1 \cdots a_n \in \mathbb{A}^\star \ : \ \left(\begin{array}{c} a_1 = a_{n-1} \wedge a_2 = a_n \wedge \\ \forall 2 \leqslant i < n-1.\ a_1 \neq a_i \wedge a_2 \neq a_{i+1} \end{array}\right)\right\}$$

$$L_2 = [|a|b(|c)^* ab]_\alpha$$

Alternation

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Definition (*Boolean Formulae*)

Let X be a set of *atoms*. Then, $\mathcal{B}_{\mathsf{n}}(X)$ denotes the set of *Boolean formulae over X* defined by the grammar

$$\varphi, \psi ::= \top \mid \bot \mid x \mid \neg\varphi \mid \varphi \vee \psi \mid \varphi \wedge \psi. \qquad (x \in X)$$

Denote by $\mathcal{B}_{+}(X)$ the subset of *positive Boolean formulae over X*, i.e. formulae that do not contain any negation.

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» If X is a nominal set, then we regard $\mathcal{B}_{\mathsf{n}}(X)$ and $\mathcal{B}_{+}(X)$ also as nominal sets with the obvious group action.

Definition (*RANA*)

A *regular alternating nominal automaton* (*RANA*) $A = (Q, \delta, q_0)$ consists of:

- an orbit-finite set Q specifying *states*;
- an equivariant *initial state* $q_0 \in Q$; and
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A RANA is *positive* if the transition function corestricts to $\mathcal{B}_{+}(1 + \mathbb{A} \times Q + [\mathbb{A}]Q)$.

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Notation

- Denote the unique atom in 1 by ε.
- Denote the atoms (a, q) and $\langle a \rangle q$ by $\Diamond_a q$ and $\Diamond_{|a} q$, respectively.

Definition (*Semantics*)

Define $w \models \varphi$ for $w \in \overline{\mathbb{A}}^\star$, and $\varphi \in \mathcal{B}_\mathsf{n}(1 + \mathbb{A} \times Q + [\mathbb{A}]Q)$ recursively by:

- $\vee$, $\wedge$, $\neg$, $\top$, and $\bot$ have the conventional interpretation.
- The interpretation of atoms $x \in \mathbb{A} \times Q + [\mathbb{A}]Q$ is given by the following clauses:

$$
\begin{aligned}
w \models \varepsilon &:\Longleftrightarrow\ w = \varepsilon \\
w \models \Diamond_a q &:\Longleftrightarrow\ \exists v \in \overline{\mathbb{A}}^\star.\ w = av \text{ and } v \models \delta(q) \\
w \models \Diamond_{|a} q &:\Longleftrightarrow\ \exists v, v' \in \overline{\mathbb{A}}^\star,\ b, c \in \mathbb{A},\ q' \in Q.\ w = {|bv} \equiv_\alpha {|cv'}, \\
&\qquad \langle a \rangle q = \langle c \rangle q',\ \text{and } v' \models \delta(q')
\end{aligned}
$$

Define the accepted languages as follows:

- **Literal Language:** $L_0(A) := \left\{ w \in \overline{\mathbb{A}}^\star\ :\ w \text{ is closed and } q_0 \text{ accepts } w \right\}$.
- **Bar Language:** $L_\alpha(A) := L_0(A)/\equiv_\alpha$.

RANA (Small Remarks)

- Our choice for transitions functions is deliberate:
  ($\delta\colon Q \to \mathcal{B}_{\mathsf{n}}(1 + \mathbb{A} \times Q + [\mathbb{A}]Q)$ instead of final states & $\delta\colon Q \times \overline{\mathbb{A}} \to \mathcal{B}_{\mathsf{n}}(Q)$)
  - Encoding transition 'letters' into formulae makes proofs and constructions easier later on. Additionally, there'd be multiple extra conditions on δ.
  - Encoding 'finality' into formulae makes transition formulae more align to their logical counterpart Bar-μTL and our modalities match more closely to their logical counterparts.
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- Why do we need a possibility of α-renaming bar strings for 'bar-modalities' $\Diamond_{|a}$?

$$w \models \Diamond_{|a} q :\Longleftrightarrow \exists v, v' \in \overline{\mathbb{A}}^{\star},\ b, c \in \mathbb{A},\ q' \in Q.\ w = |bv \equiv_\alpha |cv',\\ \langle a \rangle q = \langle c \rangle q',\ \text{and}\ v' \models \delta(q')$$

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  - Otherwise, negation *is not* α-invariant! (Example at blackboard)
  - This would contradict the expected complementation procedure!

Results I: Equivalence of Models

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Equivalence of Models

We showed that negation does not matter for expresivity of RANAs, in more detail:

Theorem (*Equivalence* **)**

Positive RANAs accept the same bar languages as ordinary RANAs do.

Hence, also the same languages under the local/global freshness semantics.

Explicit-Dual RANAs

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Definition (*Dualizable Boolean Formulae* **)**

Let X be a (nominal) set of atoms.
Put $\mathcal{B}_{\mathsf{d}}(X) := \mathcal{B}_{+}(X \cup X_{\mathsf{d}})$ with $X_{\mathsf{d}} = \left\{ x^{\mathsf{d}} \ : \ x \in X \right\}$ as a copy of X.

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Definition (***Explicit-Dual RANA*** **)**

An explicit-dual RANA $A = (Q, \delta, q_0)$ is defined like a RANA but with an equivariant transition function $\delta\colon\ Q \to \mathcal{B}_{\mathsf{d}}(1 + \mathbb{A} \times Q + [\mathbb{A}]Q)$.
We denote the copy $(\Diamond_\alpha q)^{\mathsf{d}}$ of atoms by $\Box_\alpha q$.

The additional atoms are interpreted as follows:

$$w \models \varepsilon^{\mathsf{d}} :\Longleftrightarrow\ w \neq \varepsilon$$

$$w \models \Box_a q :\Longleftrightarrow\ \forall v \in \overline{\mathbb{A}}^{\star}.\ w = av \implies v \models \delta(q)$$

$$w \models \Box_{|a} q :\Longleftrightarrow\ \forall b \in \mathbb{A}, v \in \overline{\mathbb{A}}^{\star}. w = |bv \implies w \models \Diamond_{|a} q.$$

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For every ordinary RANA A, there is an explicit-dual RANA A^{d} that accepts the *same* literal language, has twice as many orbits and the same degree as A.

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- q_{n} in A^{d} accepts the complement of q. Herefore, we 'negate' the transition formula and convert it as above.
- Acceptance is then shown easily by double induction (word length and size of transition formulae in NNF).

Equivalence (Explicit-Dual to Positive)

Proposition (*Explicit-Dual to Positive*)

For every explicit-dual RANA A of degree k and with n orbits, there is a positive RANA A^+ that accepts the *same* literal language, has degree $2k+1$, and at most $n \cdot (k+2) \cdot (2k+1)^{2k+1} + 1$ orbits.

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Given a bar string $w \in \overline{\mathbb{A}}^\star$ and a formula $\varphi \in \mathcal{B}_\mathsf{d}(1 + \mathbb{A} \times Q + [\mathbb{A}]Q)$, a *free* name $a \in \mathsf{FN}(w)$ is an *escape letter for w at φ* if the satisfaction $w \models \varphi$ 'can end' at

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- Intuition: Processing of the input word ends immediately!

...but input is still accepted!

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- We change transition formulae accordingly and verify the equivalence of languages by induction.

Results II: Equivalence to Bar-μTL

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Bar formulae of Bar-μTL are defined by the grammar

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- The semantics of bar formulae is defined like the semantics for transition formulae.

Theorem (***Equivalence*** **)**

For every bar formula φ, there is an explicit-dual RANA A_φ accepting the literal language of φ: $L_0(A) = \{w \in \mathsf{bs}(\emptyset) \,:\, w \models \varphi\}$.

$\rightsquigarrow$ makes use of the Fisher–Ladner closure of φ

Results III: De-Alternation

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Motivation

We have seen earlier that renaming is *necessary* for negation to be α-invariant. This *non-acceptance* of some α-equivalent bar strings w/o renaming was previously (ERNNAs/RNNAs) ameliorated by the use of *name-dropping*.

Definition (***Restricted Semantics*** **)**

Let $A = (Q, \delta, q_0)$ be a positive RANA. We define the *restricted satisfaction* $w \models^{\mathsf{r}} \varphi$ just as $\models$ for ε, $\Diamond_a$-modalities ($a \in \mathbb{A}$) and

$$w \models^{\mathsf{r}} \Diamond_{|a} q :\iff \exists v \in \overline{\mathbb{A}}^{\star}, b \in \mathbb{A}, q' \in Q.\ w = {|b}v, \langle a \rangle q = \langle b \rangle q' \text{ and } v \models^{\mathsf{r}} \delta(q').$$

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Theorem (*Name-Dropping for RANAs*)

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- Thus, we can restrict ourself to the restricted semantics whenever necessary.

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- We restrict the power-set construction to sets of at *most* size $n \cdot k!$.
- The resulting ERNNA has a degree of at most $n \cdot k \cdot k!$ and a number of orbits that is at most singly exponential in n and doubly exponential in k.

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Every RANA with n orbits and degree k can be de-alternated into an ERNNA (RNNA with one single $\top$-state).

Under the local freshness semantics, RANAs can be completely de-alternated into RNNAs.

- A full de-alternation to RNNAs (w/o the $\top$-state) is impossible.

  (Example at blackboard)

- Similarly, the naïve power-set construction is impossible.

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Results IV: Finitisation & Model-Checking

Friedrich-Alexander-Universität
Faculty of Engineering

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- The semantics looks at bar strings and split them up into a pre-word and a suffix, where the pre-word is '*read up to α-equivalence*' by the Bar-AFA. If it results in just $\top$'s, any suffix may be added.

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- If A is a bar AFA with alphabet size k and n states, the AFA accepting $L_{\mathsf{AFA}}(A) \cap \mathsf{bs}(\emptyset)$ has alphabet size k and at most $n + 2^{k/2} + 1$ states.

Remark (*Complexities*)

Given any RANA of degree k and with n orbits, its equivalent name-dropping modification has at most $(2 \cdot n \cdot (k+2)+1) \cdot 2^{(2k+1)\cdot \log(4k+2)}$ orbits and a degree of $2k+1$.

Its de-alternation has a degree that is linear in n and exponential in k as well as a number of orbits that is exponential in n and doubly exponential in k.

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Inclusion-Checking for RANAs is decidable in ExpSpace:

$\leadsto$ space linear in the number of both orbits and exponential in the maximum degree of both RANAs.

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Its de-alternation has a degree that is linear in n and exponential in k as well as a number of orbits that is exponential in n and doubly exponential in k.

Theorem (*Inclusion-Checking under Local Freshness*)

The inclusion problem for RANAs under local freshness is decidable in 2ExpSpace:

$\rightsquigarrow$ space exponential in both the number of orbits and the degree of both RANAs.

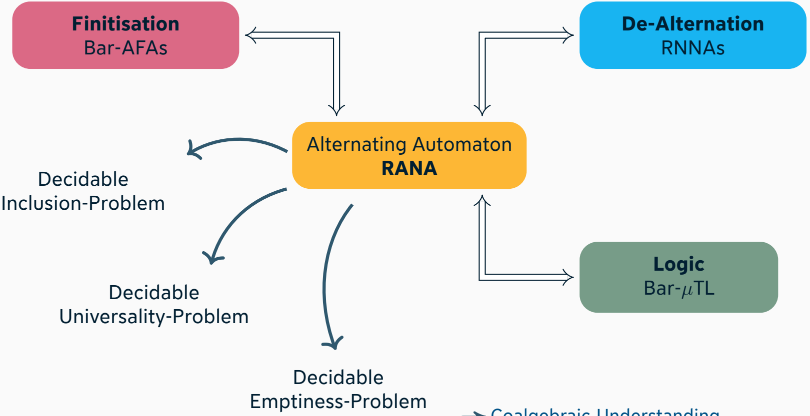
⋙ For local freshness, we need to de-alternate completely!

(Example at blackboard)

Conclusion

Conclusion & Future Work

- We looked at a variant of alternating automaton for data languages with inherent name binding, and found many nice properties:



- There are still open problems left:
  - Coalgebraic Understanding
  - Residuality/Learning RANAs
  - Extension to ω-Words

Questions?

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