Monad-Based Programming WS 2021

Assignment 4

Deadline for solutions: 23.12.2021

Exercise 1 Going Abstract (7 Points)

Recall the proof assistant exercise from the previous assignment. Your task here is to adapt your previous implementation in such a way that the main program has the following type:

```
(MonadError e m, MonadIO m, MonadState s m) => m ()
```

for suitably chosen `e`, `s` and `m`. That is, you must rewrite your code in such a way that it **only** uses the effects provided by the indicated type classes. You then need to come up with a concrete instance of `m`, in order to run your code. The facilities of the indicated type classes must be used in the following way:

- `MonadError` (from `Control.Monad.Error.Class`) is used to throw errors related to erroneous user input or inapplicability of rules
- `MonadIO` (from `Control.Monad.IO.Class`) is used to interact with the user.
- `MonadState` (from `Control.Monad.State.Class`) is used to store read and update the current proof state (list of goals).

The target instance of `m` must be obtained by combining monad transformers for state, exceptions and I/O.

Note that there is a certain freedom in defining such an instance, related to the fact that applying state transformers in different order need not produce the same result (!) For example, transforming the state monad with the exception transformer is not the same as transforming the exception monad with the state transformer. Incidentally, this is one of the reasons, to use the above abstract type classes instead of concrete monads and monad transformers. More generally, we thus following the well-known programming principle of *separating interface from implementation.*

Exercise 2 `curry` and `uncurry` (7 Points)

Given two complete partial orders A and B, let B^A be the space of continuous functions from A to B, as defined at the lecture. Recall that B^A is a partial order under the pointwise extension from B, i.e. $f \sqsubseteq g$ for $f, g \in B^A$ if $f(x) \sqsubseteq g(x)$ for all $x \in A$.

- Show that B^A are complete partial orders, if A and B are;
- Show that `curry` : $A^{C\times B} \to (A^B)^C$ and `uncurry` : $(A^B)^C \to A^{C\times B}$ are monotone and continuous (you can use without a proof that $A \times B$ is a complete partial order, if so are A and B, and the least upper bound of a sequence $(x_1, y_1) \sqsubseteq (x_2, y_2) \sqsubseteq \ldots$ is computed as $(\bigsqcup_i x_i, \bigsqcup_i y_i)$).

Exercise 3 Parallel Or (6 Points)

Recall the *parallel-or* operator from the lecture:

$$\begin{aligned}
por :&\ \mathsf{Bool}_\perp \times \mathsf{Bool}_\perp \to \mathsf{Bool}_\perp \\
por(\lfloor \mathsf{True} \rfloor, x) &= \lfloor \mathsf{True} \rfloor \\
por(x, \lfloor \mathsf{True} \rfloor) &= \lfloor \mathsf{True} \rfloor \\
por(\lfloor \mathsf{False} \rfloor, \lfloor \mathsf{False} \rfloor) &= \lfloor \mathsf{False} \rfloor \\
por(x, y) &= \perp, \qquad\qquad \text{otherwise}
\end{aligned}$$

Prove that it is continuous.