Learning Automata with Name Allocation

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Research Seminar

Chair for Computer Science 8 (Theoretical Computer Science)
Friedrich-Alexander-Universität Erlangen-Nürnberg

Friedrich-Alexander-Universität
Faculty of Engineering

Motivation (Register Automata & Data Languages)

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$\lambda a.(\lambda b.)^\star a$
(using shadowing)

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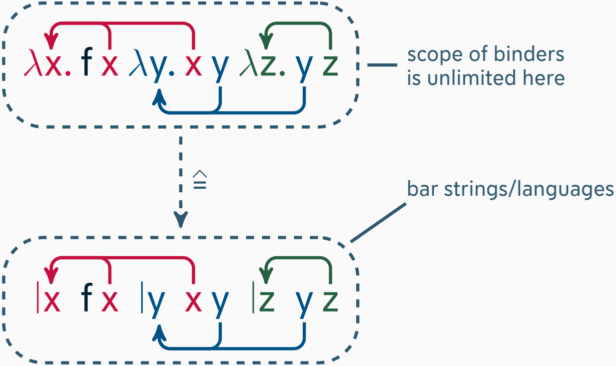

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$$|x\ f\ x\ |y\ x\ y\ |z\ y\ z$$

bar strings/languages

admits an α-equiv.

$\equiv_\alpha$

$$|a\ f\ a\ |b\ a\ b\ |a\ b\ a$$

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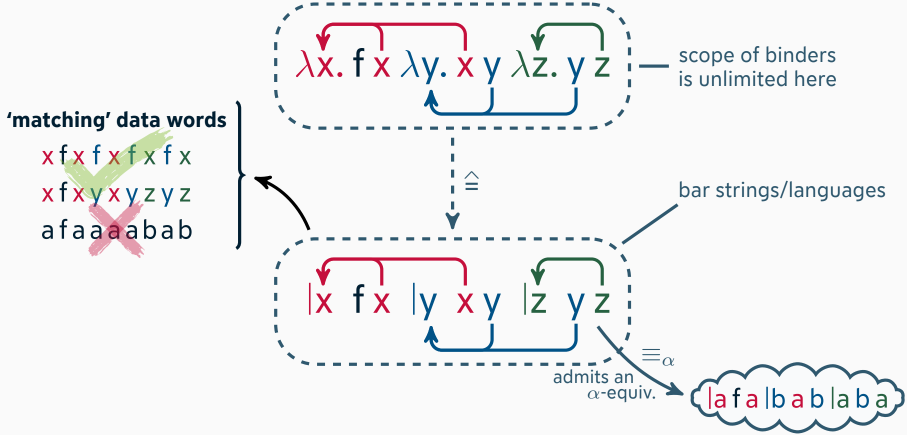

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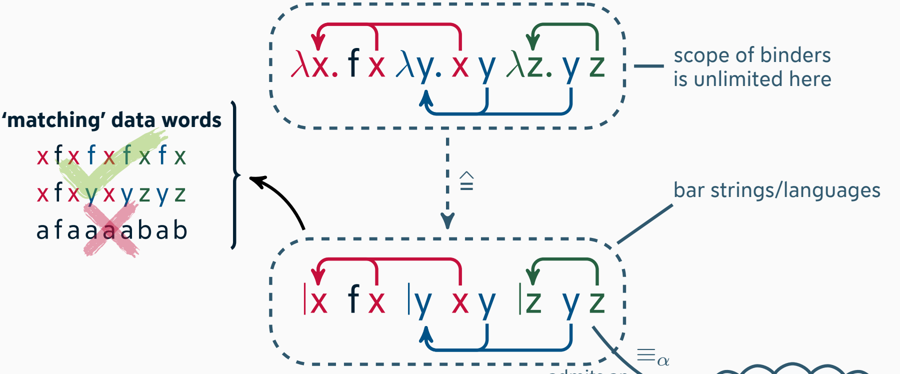

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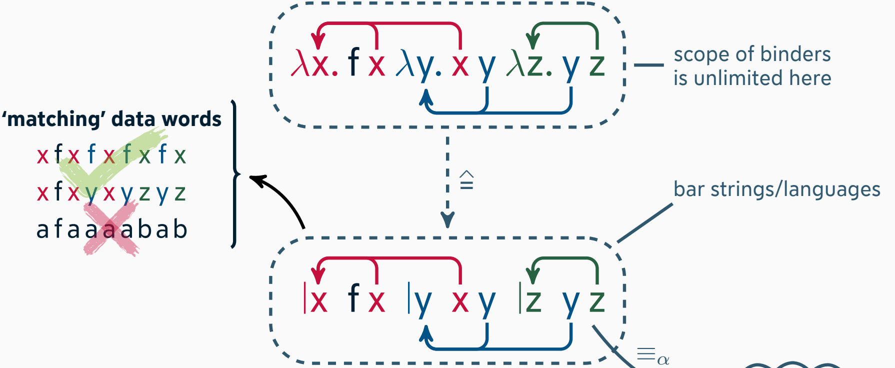

- 'Match' data words by taking any representative without bars.
- **Our Problem:** How do we get the neccessary models for model checking from a black-box system?

Models for Bar Languages

» We consider classical automata over *finite* subalphabets $\overline{\mathbb{A}}_0 \subseteq_\mathsf{f} \overline{\mathbb{A}}$:

Definition (*Bar DFA*)

A bar DFA $\mathscr{A}$ is a DFA over a finite alphabet $\overline{\mathbb{A}}_0 \subseteq_\mathsf{f} \overline{\mathbb{A}}$.

Its *bar language* $L_\alpha(\mathscr{A}) = \left\{ w \in \overline{\mathbb{A}}^\star \,:\, w \equiv_\alpha w' \in L(\mathscr{A}) \right\}$ consists of all representatives of its α-equivalence classes.

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- Expressivity (data languages):
  subclass of register automata

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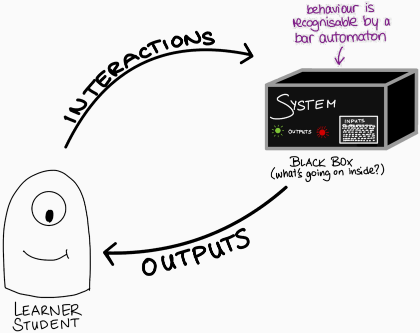
» **Task:** Infer an automaton behaving ‘identically’ to the black-box system.



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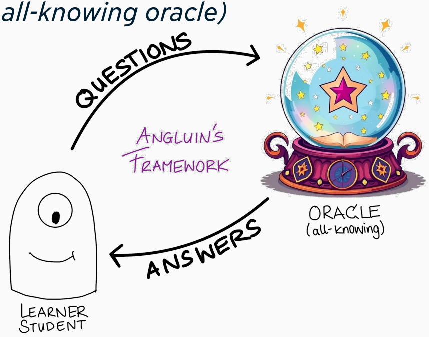

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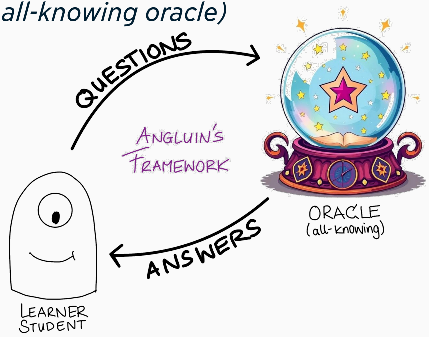

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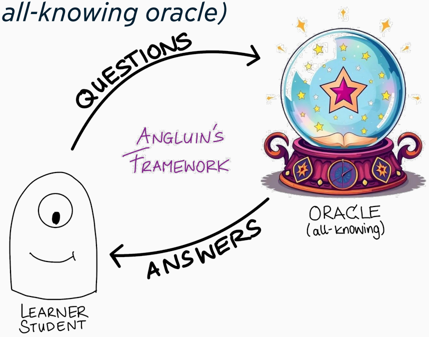

Active Automata Learning

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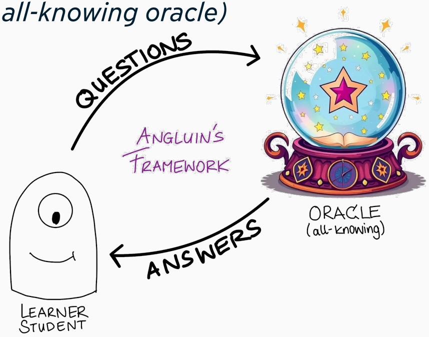
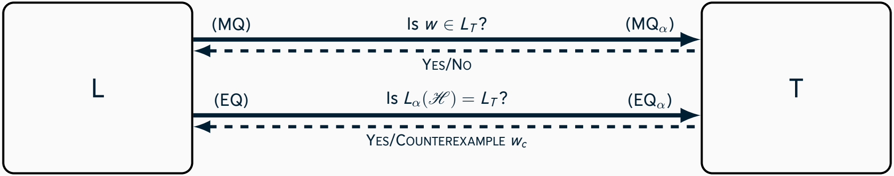


Learning Bar Automata

Question: Can't we just use classical learning algorithms for bar automata (as they are just DFA with additional semantics)?



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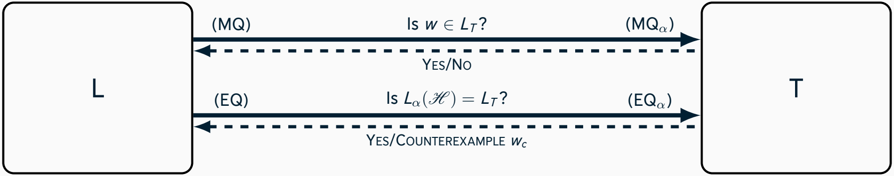

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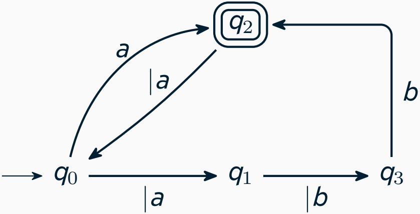

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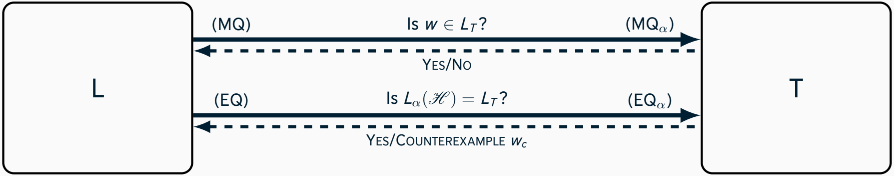

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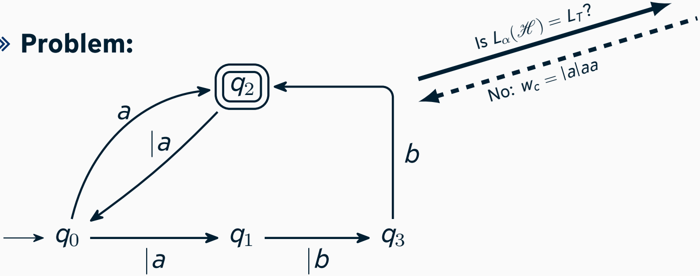

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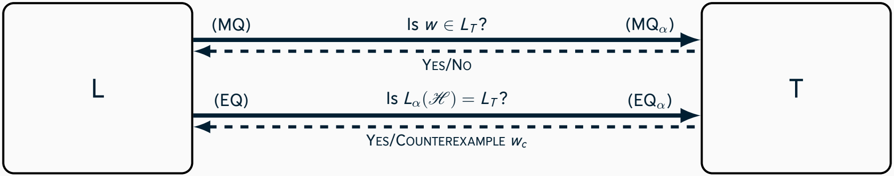

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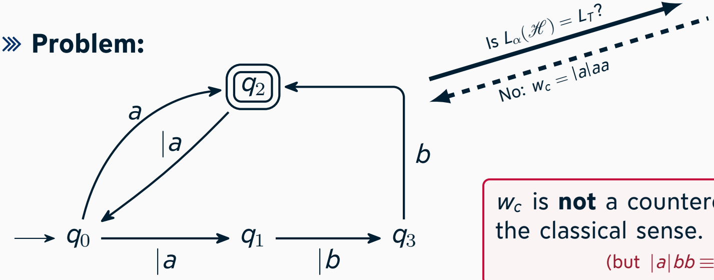
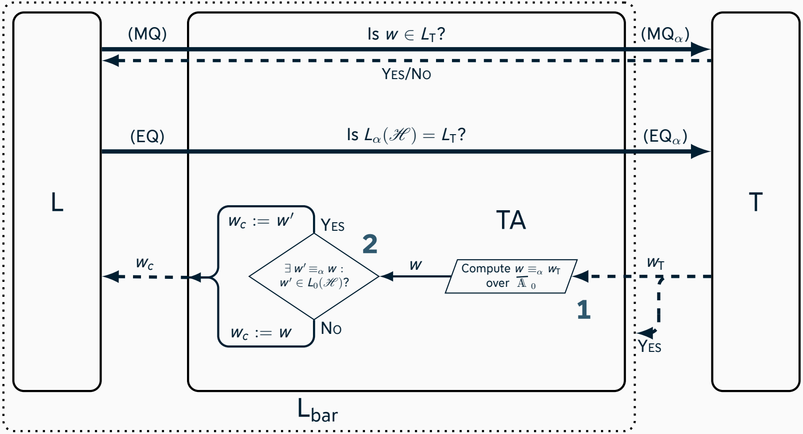


w_c is **not** a counterexample in the classical sense.

(but $|a|bb \equiv_\alpha w_c$ would be)

» Indeed, this is the only problem with the previous approach:



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Given some bar word $w_\top \in \overline{\mathbb{A}}^\star$, compute $w \equiv_\alpha w_\top$ over $\overline{\mathbb{A}}_0^\star$

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- These normal forms are *unique* (per equivalence class) and computable in *polynomial time* (with linear-logarithmic space).

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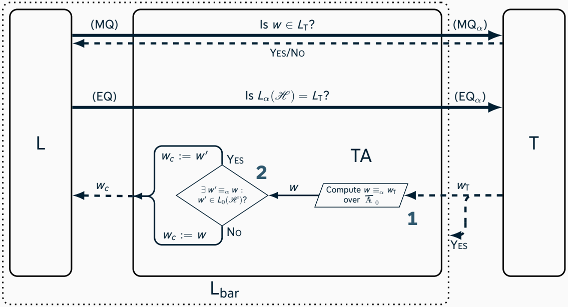
  - ... but for fixed alphabets in deterministic poly-time.

by comparing with the *closed* bar automaton.

» By solving both problems, correctness of our approach is shown.



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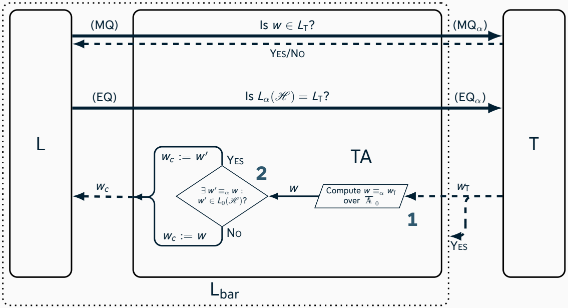

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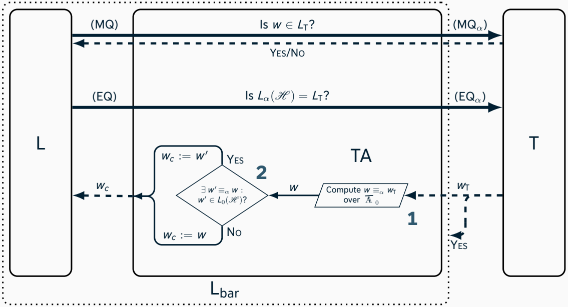

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- Implicit Assumption: $\mathrm{L_{bar}}$ knows the *number of registers* (size of alphabet) needed for L_T.

How to handle an unknown Alphabet?

Idea

If L_{bar} gets stuck, just restart anew (with an extended alphabet).

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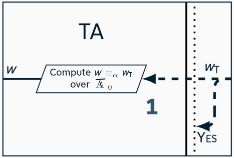

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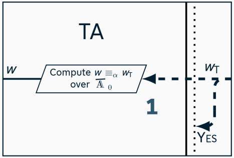

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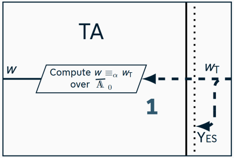

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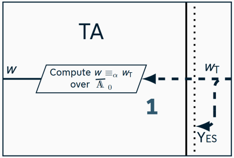

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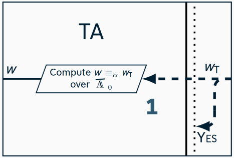

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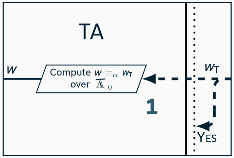

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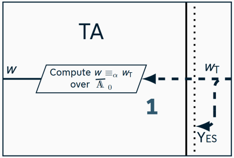

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Theorem (*Extension Complexity*)

L$_{\text{bar}}$ can infer a bar automaton using the minimal alphabet with at most as many queries as L would need (summed over all smaller cardinalities).

What about ω- and tree languages?

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Alphatic Trees

As expected, everything works out analogously:

- Normal forms are computed branchwise (and thus still in *poly-time*).

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- Next to finite word languages, we extended our approach to bar ω- and bar tree languages:

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T.CS

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- There is no suitable normal form for infinite strings.
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- Checking α-equivalence is still in poly-time, but guessing seemingly impossible.
- Expensive computation of closures is important!

Conclusion & Future Work

- Learnability of various kinds of bar languages in Angluin's framework.
- Our learner is in terms of query complexity as optimal as the underlying learner.
- Introduced efficient procedures for checking α-equivalence.

Future Work

- Is guessing possible for bar ω-languages? Can the computation of closures be removed?
- Can this approach be extended to efficiently learn data languages?
- What to do about conformance testing?

Questions?

Friedrich-Alexander-Universität
Faculty of Engineering

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