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1 Class190423

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`reverse_naive :: [a] -> [a]`

Naive list reversal

Theorem (attempt): `reverse (reverse xs) = xs`

Proof attempt:

IB: let `xs = []`, then:

```
reverse (reverse [])           { []-case }
= reverse []                   { []-case }
= []
```

IS: let `xs = x : xs'`, then

```
reverse (reverse (x : xs))     { :-case }
= reverse (reverse xs ++ [x]) { :-case }
= ...
= bummer
```

Reversal with accumulator:

`reverse_with_acc :: [a] -> [a] -> [a]`

`reverse :: [a] -> [a]`

Now, we can define `reverse` properly.

- Our goal is still: `reverse (reverse xs) = xs`
- Observe (informally): `reverse_with_acc xs ys = (reverse xs) ++ ys`

- This helps to notice that $\text{rev (rwa xs ys)} = \text{rwa ys xs}$, which is sufficient, because we can then just take $\text{ys} = []$, and then, by definition, and by $[]$ -case,

$$\text{rev (reve xs)} = \text{rev (rwa xs [])} = \text{rwa [] xs} = \text{rev xs}$$

So, let us prove $\text{rev (rwa xs ys)} = \text{rwa ys xs}$, by induction on xs

IB: $\text{xs} = [] \rightarrow$

$$\begin{aligned} & \text{rev (rwa [] ys)} \\ = & \text{rev ys} && \{ []\text{-case for rwa} \} \\ = & \text{rwa ys []} && \{ \text{def. of rev} \} \end{aligned}$$

IS: $\text{xs} = x : \text{xs}' \rightarrow$

$$\begin{aligned} & \text{rev (rwa (x : xs') ys)} \\ = & \text{rev (rwa xs' (x : ys))} && \{ \text{: -case for rwa} \} \\ = & \text{rwa (x : ys) xs'} && \{ \text{IH} \} \\ = & \text{rwa ys (x : xs')} && \{ \text{: -case for rwa} \} \end{aligned}$$