Representing Guardedness in Call-By-Value

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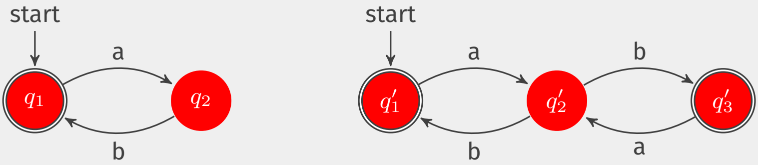
How do we know that automata



are equivalent?

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[1] A. Salomaa, Two Complete Axiom Systems for the Algebra of Regular Events, 1966

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- This only works because $x \mapsto abx + 1$ is guarded
- $x \mapsto (a+1)x + 1$ is un-guarded and has infinitely many fixpoints

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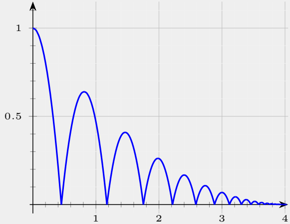
Bouncing ball is a simple Newtonian system specified by differential equation $\ddot{h} = -g$ ($g \approx 9.8$) whose solution is

$$h(t) = h_0 + v_0 t - \frac{gt^2}{2}$$

with initial values:

- $v_0 = 0$, $h_0 \neq 0$ (peak height)
- $h_0 = 0$, $v_0 \neq 0$ (zero height)



This system is progressive: every iteration consumes non-zero time (although it keeps getting smaller – Zeno behaviour)

Non-progressive (chattering) behaviour is often regarded a modelling artefact

Basic Process Algebra (BPA):

$$P, Q, \ldots := \checkmark \mid a \in A \mid P + Q \mid P \cdot Q$$

E.g. we can specify a 2-cell FIFO, storing bits:

$$\begin{aligned} B_0 &= \mathsf{in}_0.\, B_1^0 + \mathsf{in}_1.\, B_1^1 \\ B_1^i &= \mathsf{in}_0.\, B_2^{0,i} + \mathsf{in}_1.\, B_2^{1,i} + \mathsf{out}_i.\, B_0 && (i \in \{0,1\}) \\ B_2^{i,j} &= \mathsf{out}_j.\, B_1^i && (i, j \in \{0,1\}) \end{aligned}$$

Solutions are unique for guarded specifications. Otherwise not: $X = X$ has infinitely many solutions

Monads for Denotations

We can model previous examples with monads, augmented with partially defined iteration operators

$$\frac{f\colon X \to T(Y+X)}{f^{\dagger}\colon X \to TY}$$

w.r.t. a co-Cartesian category (=category with finite coproducts)

1. Automata: $TX = \mathcal{P}(A^{\star} \times X)$
2. Hybrid time: $TX = \mathbb{R}_{\geqslant 0} \times X + \bar{\mathbb{R}}_{\geqslant 0}$
3. BPA: $TX = \nu\gamma.\,\mathcal{P}_{\omega_1}(X + A \times \gamma)$ (final F-coalgebra)

Note that a monad carries information about computational effects, but not about guardedness

Guarded v.s. Unguarded

Most of time, guarded fixpoints are restrictions of unguarded ones. But the guarded ones are better behaved:

- Often unique, hence enable reasoning by coinduction
- If not unique, often computed as least fixponts
- Foundation-independent
- Simpler to define and to work with

This motivates a type discipline for propagating guardedness over structures

Iteration and Recursion

Iteration vs. Recursion

- Iteration operator:

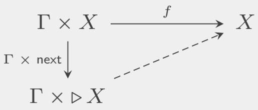
$$\frac{f\colon X \to Y + X}{f^{\dagger}\colon X \to Y}$$

- Dually: recursion operator:

$$\frac{f\colon \Gamma \times X \to X}{f_{\dagger}\colon \Gamma \to X}$$

  equivalently: $\mathsf{fix}\colon (X \to X) \to X$, e.g. in the λ-calculus

- Guarded recursion: $\mathsf{fix}\colon (\triangleright X \to X) \to X$
  - Curry-Howard counterpart of the Löb rule
  - Familiar model: topos of trees $\mathbf{Set}^{\omega^{\mathrm{op}}}$[2]
  - Notion of guardedness is representable: $f\colon \Gamma \times X \to Y$ is guarded (=contractive) iff f factors as



[2] L. Birkedal et al, First Steps in Synthetic Guarded Domain Theory: Step-Indexing in the Topos of Trees, 2011

Problem

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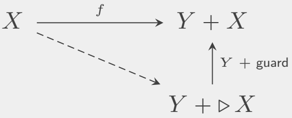
Sticking to iteration, can we generally define representable guardedness?

- Maybe (?) we need an endofunctor $\triangleright$, and then $f\colon X \to Y + X$ is guarded if it factors



- Example: $f\colon X \to (Y + X) \times \mathbb{N} \cong Y \times \mathbb{N} + X \times \mathbb{N}$ is guarded iff it factors through $(Y \times \mathbb{N} + X \times \mathsf{suc})$
- However, e.g. $f\colon X \to (Y + X)^{\star}$ should be guarded if in every $f(x) = [e_1, \ldots, e_n]$ every $e_n \in X$ is preceded by some $e_k \in Y$

$\Rightarrow$ "$\triangleright$" may depend both on X and on Y

Approach

- An identity-on-object functor $J\colon \mathbf{V} \to \mathbf{C}$ has a right adjoint iff
  - $\mathbf{C}$ is isomorphic to Kleisli category of a monad on $\mathbf{V}$[3]
  - all presheaves $\mathbf{C}(J, A)\colon \mathbf{V}^{\mathrm{op}} \to \mathbf{Set}$ are representable
- Fine-grain call-by-value[4] was interpreted over Freyd categories, which are certain identity-on-object functors $J\colon \mathbf{V} \to \mathbf{C}$ where
  - $\mathbf{V}$ is a category of values
  - $\mathbf{C}$ is a category of computations

  All $J(- \times A)\colon \mathbf{V} \to \mathbf{C}$ have right adjoints iff
  - $\mathbf{C}$ is isomorphic to a Kleisli category of a strong monad T, and all Kleisli exponentials B^{TA} exist
  - all presheaves $\mathbf{C}(J(- \times A), B)\colon \mathbf{V}^{\mathrm{op}} \to \mathbf{Set}$ are representable
- Here: representability of guardedness in fine-grain call-by value

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[3] D. Schumacher, Minimale und Maximale Tripelerzeugende und eine Bemerkung zur Tripelbarkeit, 1969

[4] P. Levy, J. Power, H. Thielecke, Modelling Environments in Call-By-Value Programming Languages, 2002

Representing Guardedness

Abstract (coCartesian) Guardedness

A guardedness predicate identifies for all objects X, Y, Z guarded morphisms $\mathbf{C}_\bullet(X, Y, Z) \subseteq \mathbf{C}(X, Y + Z)$, such that

$$\textbf{(trv}_+\textbf{)}\ \frac{f\colon X \to Y}{\mathsf{inl}\, f\colon X \to Y \rangle Z} \qquad \textbf{(par}_+\textbf{)}\ \frac{f\colon X \to V \rangle W \quad g\colon Y \to V \rangle W}{[f, g]\colon X + Y \to V \rangle W}$$

$$\textbf{(cmp}_+\textbf{)}\ \frac{f\colon X \to Y \rangle Z \qquad g\colon Y \to V \rangle W \qquad h\colon Z \to V + W}{[g, h]\, f\colon X \to V \rangle W}$$

where $f\colon X \to Y \rangle Z$ means $f \in \mathbf{C}_\bullet(X, Y, Z)$

- A category with a guardedness predicate is called guarded
- A monad is guarded if its Kleisli category is guarded

Examples

- $f\colon X \to \mathcal{P}(A^\star \times (Y + Z))$ is guarded if it factors through

$$\begin{aligned}\mathcal{P}(A^\star \times Y + A^+ \times Z) &\hookrightarrow \mathcal{P}(A^\star \times Y + A^\star \times Z) \\ &\cong \mathcal{P}(A^\star \times (Y + Z))\end{aligned}$$

- $f\colon X \to \mathbb{R}_{\geqslant 0} \times (Y + Z) + \bar{\mathbb{R}}_{\geqslant 0}$ is guarded if it factors through

$$\begin{aligned}\mathbb{R}_{\geqslant 0} \times Y + \mathbb{R}_{>0} \times Z + \bar{\mathbb{R}}_{\geqslant 0} &\hookrightarrow \mathbb{R}_{\geqslant 0} \times Y + \mathbb{R}_{\geqslant 0} \times Z + \bar{\mathbb{R}}_{\geqslant 0} \\ &\cong \mathbb{R}_{\geqslant 0} \times (Y + Z) + \bar{\mathbb{R}}_{\geqslant 0}\end{aligned}$$

- $f\colon X \to \nu\gamma.\, T((Y + Z) + H\gamma)$ is guarded if it factors through

$$\begin{aligned}T(Y + H(\nu\gamma.\, \ldots)) &\hookrightarrow T((Y + Z) + H(\nu\gamma.\, \ldots)) \\ &\cong \nu\gamma.\, T((Y + Z) + H\gamma)\end{aligned}$$

Call-by-Value with Effects

Very Simple Metalanguage (VSML)

- Sorts $A, B, C, \ldots$
- Signatures Σ_v, Σ_c of pure and effectful programs $f\colon A \to B$
- Semantics of (Σ_v, Σ_c) w.r.t. identity-on-objects functor $J\colon \mathbf{V} \to \mathbf{C}$:
  - an object $[\![A]\!] \in |\mathbf{V}|$ to each sort A
  - a morphism $[\![f]\!] \in \mathbf{V}([\![A]\!], [\![B]\!])$ to each $f\colon A \to B \in \Sigma_v$
  - a morphism $[\![f]\!] \in \mathbf{C}([\![A]\!], [\![B]\!])$ to each $f\colon A \to B \in \Sigma_c$
- Terms in single-variable (!) context:

$$\frac{f\colon A \to B \in \Sigma_v \quad \Gamma \vdash_{\mathsf{v}} v\colon A}{\Gamma \vdash_{\mathsf{v}} f(v)\colon B} \qquad \frac{f\colon A \to B \in \Sigma_c \quad \Gamma \vdash_{\mathsf{v}} v\colon A}{\Gamma \vdash_{\mathsf{c}} f(v)\colon B}$$

$$\frac{}{x\colon A \vdash_{\mathsf{v}} x\colon A} \qquad \frac{\Gamma \vdash_{\mathsf{v}} v\colon A}{\Gamma \vdash_{\mathsf{c}} \mathsf{return}\, v\colon A} \qquad \frac{\Gamma \vdash_{\mathsf{c}} p\colon A \quad x\colon A \vdash_{\mathsf{c}} q\colon B}{\Gamma \vdash_{\mathsf{c}} x \leftarrow p;\, q\colon B}$$

- $[\![-]\!]$ extends easily

The fine-grain call-by-value (FGCBV) is obtained by enabling multivariable contexts $\Gamma = (x_1 \colon A_1, \ldots, x_n \colon A_n)$, e.g. variable term formation:

$$\frac{}{x_1 \colon A_1, \ldots, x_n \colon A_n \vdash_{\mathsf{v}} x_i \colon A_i}$$

FGCBV can be interpreted over a Freyd category:

- $\mathbf{V}$ is a category with finite products
- action $\mathbf{V} \times \mathbf{C} \to \mathbf{C}$ of $\mathbf{V}$ on $\mathbf{C}$
- $J \colon \mathbf{V} \to \mathbf{C}$ is an identity-on-objects functor, preserving the action

- Originally, Moggi[5] interpreted call-by-value over strong monads
- T is strong if it comes with strength

$$\tau \colon X \times TY \to T(X \times Y)$$

which satisfies a number of coherence conditions
- We then can interpret

$$\frac{f := [\![\Gamma \vdash_{\mathsf{c}} p \colon A]\!] \quad g := [\![\Gamma, x \colon A \vdash_{\mathsf{c}} q \colon B]\!]}{[\![\Gamma \vdash_{\mathsf{c}} x \leftarrow p;\, q \colon B]\!] \colon [\![\Gamma]\!] \xrightarrow{\langle \Gamma, f \rangle} [\![\Gamma]\!] \times T[\![A]\!] \xrightarrow{\tau} T[\![\Gamma \times A]\!] \xrightarrow{g^\star} T[\![B]\!]}$$

[5] E. Moggi, Notions of Computation and Monads, 1991

If we want to implement higher order:

$$\frac{\Gamma, x\colon A \vdash_{\mathsf{c}} p\colon B}{\Gamma \vdash_{\mathsf{v}} \lambda x.\, p\colon A \to B} \qquad \frac{\Gamma \vdash_{\mathsf{v}} f\colon A \to B \qquad \Gamma \vdash_{\mathsf{v}} v\colon A}{\Gamma \vdash_{\mathsf{c}} f\, v\colon B}$$

we need to have a semantics $[\![A \to B]\!] = U([\![A]\!], [\![B]\!])$, such that

$$\mathbf{C}(J(X \times A), B) \cong \mathbf{V}(X, U(A, B))$$

naturally in A

Theorem ([6])

The following are equivalent:

- $\mathbf{C}(J(X \times A), B) \cong \mathbf{V}(X, U(A, B))$ *for some* $U\colon \mathbf{V} \times \mathbf{C} \to \mathbf{V}$, *naturally in* A
- *Presheaves* $\mathbf{C}(J(X \times (\text{–})), B)\colon \mathbf{V}^{\mathsf{op}} \to \mathbf{Set}$ *are representable*
- $\mathbf{C}$ *is isomorphic to a Kleisli category of a strong monad* $\mathbf{T}$ *on* $\mathbf{V}$ *and all exponentials* $(TB)^A$ *exist*

[6] Essentially: P. Levy, J. Power, H. Thielecke, Modelling Environments in Call-By-Value Programming Languages, 2002

(Non-)Strong Monads

If we do not care about strength, we have a simpler characterization

Theorem (Schumacher)

Given id-on-objects functor $J\colon \mathbf{V} \to \mathbf{C}$, the following are equivalent:

- *J is a left adjoint*
- *Presheaves $\mathbf{C}(J(-), B)\colon \mathbf{V}^{\mathrm{op}} \to \mathbf{Set}$ are representable*
- *$\mathbf{C}$ is isomorphic to a Kleisli category of a monad*

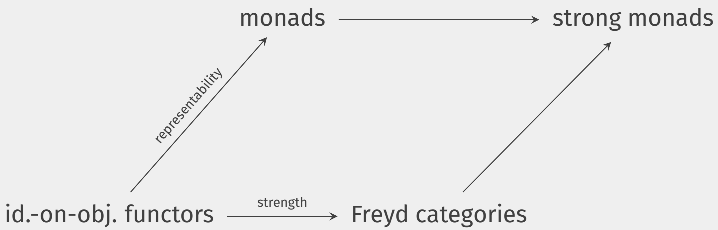
and then:

Theorem

Given a Freyd category $J\colon \mathbf{V} \to \mathbf{C}$, the following are equivalent:

- *J is a left adjoint*
- *Presheaves $\mathbf{C}(J(-), B)\colon \mathbf{V}^{\mathrm{op}} \to \mathbf{Set}$ are representable*
- *$\mathbf{C}$ is isomorphic to a Kleisli category of a strong monad*

Intermediate Summary



Call-by-Value Meets Guardedness

Representing Guardedness

Definition

Given $J\colon \mathbf{V} \to \mathbf{C}$, as before and guarded $\mathbf{C}$, call the guardedness predicate $\mathbf{C}_\bullet$ **(J-)representable** if for all $A, B \in |\mathbf{C}|$ the presheaves

$$\mathbf{C}_\bullet(J(-), A, B)\colon \mathbf{V}^{\mathsf{op}} \to \mathbf{Set}$$

are representable

Note that $\mathbf{C}_\bullet(X, A, \emptyset) \cong \mathbf{C}(X, A)$, hence

Lemma

If $\mathbf{C}_\bullet$ is representable, J is a left adjoint. In this case, $\mathbf{C}$ is a Kleisli category of some monad on $\mathbf{V}$

Guarded Parametrized Monads

Recall that a bifunctor $\#\colon \mathbf{V}\times\mathbf{V}\to\mathbf{V}$ is a parametrized monad[7] if

- Every $(-)\#X$ is a monad
- Every $(-)\#f$ is a monad morphism

Definition

A guarded parametrized monad on a symmetric monoidal $(\mathbf{V},\otimes,I,\rho,\lambda,\alpha,\gamma)$ consists of a bifunctor $\#\colon \mathbf{V}\times\mathbf{V}\to\mathbf{V}$, natural transformations

$$\eta\colon\ A\to A\#I$$

$$\upsilon\colon\ A\#(B\otimes C)\to(A\otimes B)\#C \qquad \xi\colon\ (A\#B)\#C\to A\#(B\otimes C)$$

$$\chi\colon\ A\#B\otimes C\#D\to(A\otimes C)\#(B\otimes D) \qquad \zeta\colon\ A\#(B\#C)\to A\#(B\otimes C)$$

plus a bunch of commutative diagrams

- Intuition: in $X\#Y$, Y is the guarded part, X is (possibly) unguarded part
- Coherence property: if $f,g\colon \mathcal{E}_1\to\mathcal{E}_2\#\mathcal{E}_2'$ and
  - f, g are made of η, ϵ, υ, ξ, ζ, ρ, λ, α, γ, id, $\otimes$, $\#$
  - object letters do not repeat either in $\mathcal{E}_1$ or in $\mathcal{E}_2\#\mathcal{E}_2'$
  - $\mathcal{E}_2$ and $\mathcal{E}_2'$ do not contain $\#$

  then $f=g$

[7] T. Uustalu, Generalizing Substitution, 2003

Main Theorem

Theorem

Given co-Cartesian $\mathbf{V}$ *and an identity-on-object functor* $J\colon \mathbf{V} \to \mathbf{C}$ *strictly preserving coproducts,* $\mathbf{C}$ *is guarded and* $\mathbf{C}_\bullet$ *is representable iff*

- $\mathbf{C} \cong \mathbf{V}_{-\#0}$ *for a guarded parametrized monad* $(\#, \eta, \upsilon, \chi, \xi, \zeta)$
- *the compositions*

$$X \# Y \cong X \# (Y + \emptyset) \xrightarrow{\upsilon_{X,Y,\emptyset}} (X + Y) \# \emptyset$$

  are all monic and
- $f\colon X \to Y \rangle Z$ *iff* f *factors through* $Y \# (Z + 0) \xrightarrow{\upsilon} (Y + Z) \# \emptyset$

Analogously, representability of

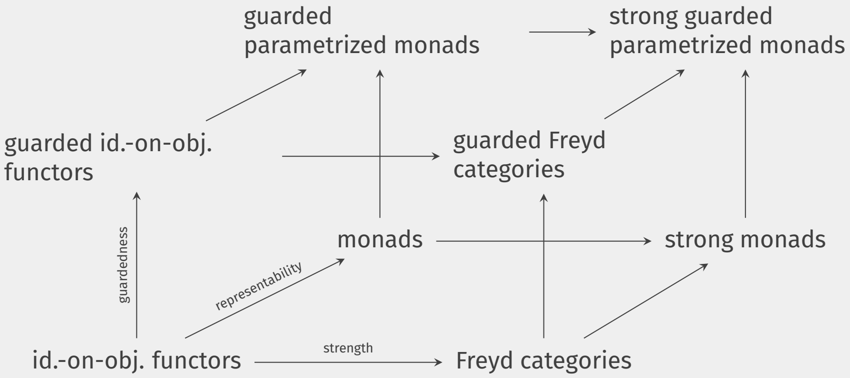
$$\mathbf{C}_\bullet(J(- \times A), B, C)\colon \mathbf{V}^{\mathsf{op}} \to \mathbf{Set}$$

produces strong guarded parametrized monads

Examples

- Least guardedness: $X \# Y = TX$
- Greatest guardedness: $X \# Y = T(X + Y)$ (exception transformer)
- Automata: $X \# Y = \mathcal{P}(A^\star \times X + A^+ \times Y)$
- Hybrid systems: $X \# Y = \mathbb{R}_{\geqslant 0} \times X + \mathbb{R}_{>0} \times Y + \bar{\mathbb{R}}_{\geqslant 0}$
- Generalized processes: $X \# Y = T(X + H(\nu\gamma.\, T((X + Y) + H\gamma)))$

Total Summary



Further Work

- Generalize: express quantitative information by typing, e.g. how productive is the program, how much time it consumes, etc.
- Dualize: representation of guarded recursion by comonads
  - What are instances of comonadic guarded recursion?
  - Representing recursion on casual streams/course-of-value recursion
- Implement (Haskell, Agda, Coq)
- Research:
  - Can we prove more general coherence theorem?
  - Are properly monoidal guarded parametrized monads interesting?
  - Can we characterize guardedness by unary functors (like $\triangleright$)?

Thank you for your **Attention**!

Originally, Moggi[8] interpreted call-by-value over strong monads

- A functor $F\colon \mathbf{C} \to \mathbf{D}$ between monoidal $\mathbf{C}$ and $\mathbf{D}$ is strong if there is (natural in A, B) strength $\tau_{A,B}\colon A \otimes FB \to F(A \otimes B)$, such that

$$
\begin{array}{ccc}
I \otimes FX & \cong & FX \\
{\scriptstyle\tau}\downarrow & & \| \\
F(I \otimes X) & \cong & FX
\end{array}
\qquad
\begin{array}{ccccc}
(X \otimes Y) \otimes FZ & & \xrightarrow{\quad\tau\quad} & & F((X \otimes Y) \otimes Z) \\
\wr\| & & & & \wr\| \\
X \otimes (Y \otimes FY) & \xrightarrow{X \otimes \tau} & X \otimes F(Y \otimes Z) & \xrightarrow{\tau} & F(X \otimes (Y \otimes Z))
\end{array}
$$

- A monad (T, η, μ) on $\mathbf{C}$ is strong if T is strong additionally η, μ are strong:

$$
\begin{array}{ccc}
X \otimes Y & = & X \otimes Y \\
{\scriptstyle X \otimes \eta}\downarrow & & \downarrow{\scriptstyle\eta} \\
X \otimes TY & \xrightarrow{\tau} & T(X \otimes Y)
\end{array}
\qquad
\begin{array}{ccccc}
X \otimes TTY & \xrightarrow{\tau} & T(X \otimes TY) & \xrightarrow{T\tau} & TT(X \otimes Y) \\
{\scriptstyle X \otimes \mu}\downarrow & & & & \downarrow{\scriptstyle\mu} \\
X \otimes TY & & \xrightarrow{\quad\tau\quad} & & T(X \otimes Y)
\end{array}
$$

Theorem ([9])

In monoidal closed categories, strength is equivalent to enrichment

[8] E. Moggi, Notions of Computation and Monads, 1991

[9] A. Kock, Strong Functors and Monoidal Monads, 1972

Non-Representable Guardedness

If J is not a left adjoint, guardedeness is not representable. But this is boring. Is there other counterexamples?

Theorem

Suppose, every morphism in $\mathbf{V}$ *factorizes as a regular epic, followed by a monic. Let* $\mathbf{T}$ *be a guarded monad on* $\mathbf{V}$*. Then a family of monos* $(\epsilon_{X,Y} : X \mathbin{\#} Y \hookrightarrow T(X + Y))_{X,Y \in |\mathbf{V}|}$ *extends to a guarded parametrized monad iff*

- *every* $\epsilon_{X,Y}$ *is the* ***largest*** *guarded subobject of* $T(X + Y)$
- *for every* $f : X \to T(Y + Z)$ *and a regular epic* $g : X' \to X$*, if* $f\,g$ *is guarded then* f *is guarded*

Example

In $\mathbf{Set}$, let $f : X \to Y + Z$ be guarded in Z if $\{z \in Z \mid f^{-1}(\mathsf{inr}\, z) \neq \varnothing\}$ is finite. This predicate is not Id-representable, as any $1 \hookrightarrow X \xrightarrow{\mathsf{inr}} \emptyset + X$ is guarded, but inr is not if X is infinite.