Alternating Nominal Automata with Name Allocation

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Friedrich-Alexander-Universität Erlangen-Nürnberg

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FAU Friedrich-Alexander-Universität
Faculty of Engineering

Motivation (Register Automata & Data Languages)

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'first and last user coincide and differ from any other user'

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Result — **Schröder, Kozen, Milius, Wißmann '17**

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$\lambda a.(\lambda b.)^\star a$ (using shadowing)

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- We consider data languages with explicit binders, which we see with λ-terms without parenthesis:

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scope of binders is unlimited here

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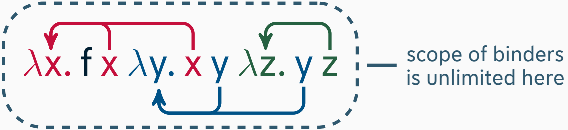

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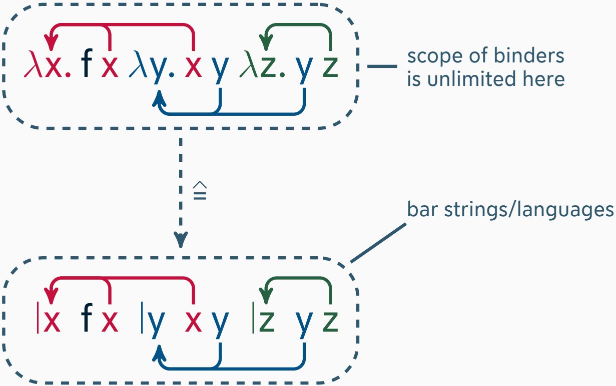

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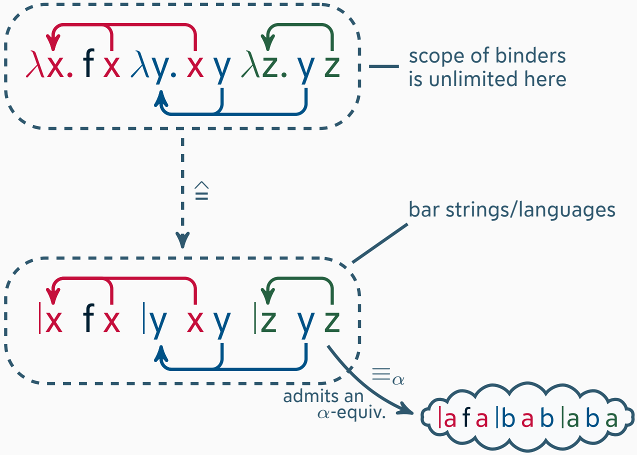

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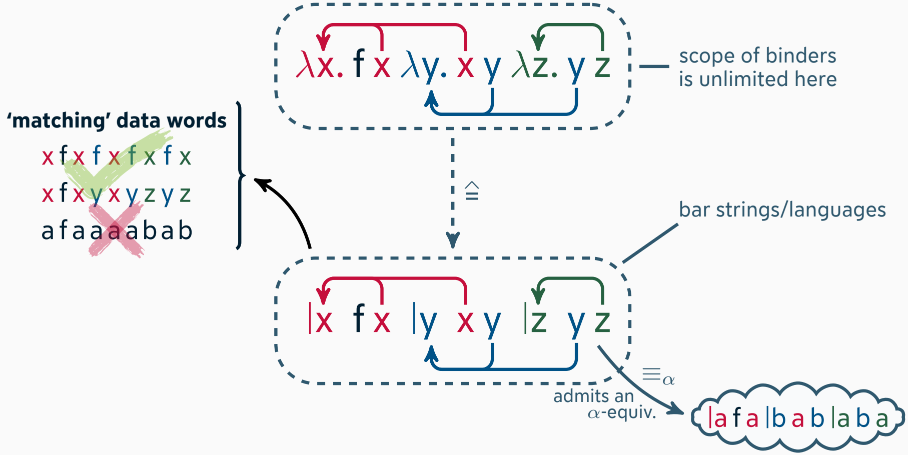

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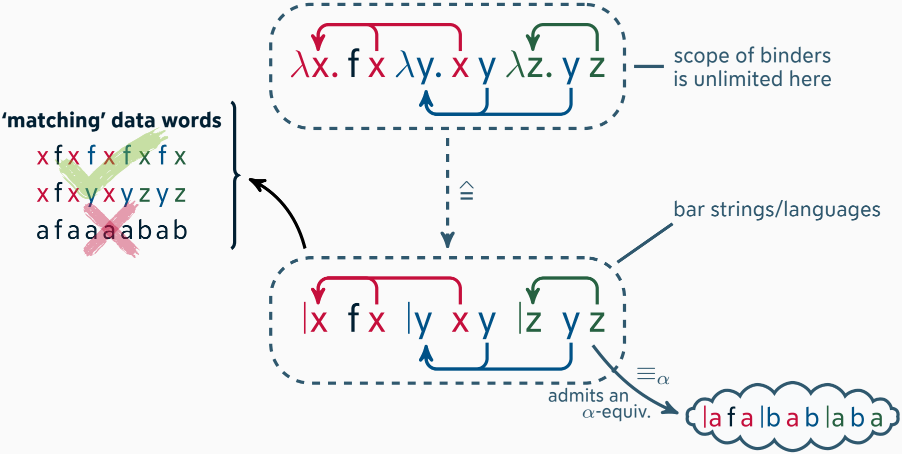

- 'Match' data words by taking any representative without bars.

Alternation For Bar Languages

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Remove de-alternation from model checking fixed-point logics over bar strings.

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Previous Approach: (Hausmann, Milius, Schröder '21)

linear-time fixed-point logic → Bar-μTL

Bar-μTL —doubly exponential blowup→ ERNNA

non-deterministic automata with $\top$-states → ERNNA

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Example of a Bar-μTL formula (over closed bar strings)

$$\varphi = \mu X.(\Diamond_{|a}(\Diamond_{|a}\top \vee \Diamond_a X \vee \varepsilon) \wedge \Diamond_{|a}\top)$$

- $\Diamond_{|a}$, $\Diamond_a$: interpreted as 'consumes the letter' (modulo α-equiv.)
- φ: formulae are interpreted over finite bar strings ($w \models \varphi$)
- μX … X: 'recursion'
- ε: interpreted as 'string is empty'

» $|a|ba \models \varphi$ » $|aa|a \models \varphi$ » $|aa|ba \not\models \varphi$

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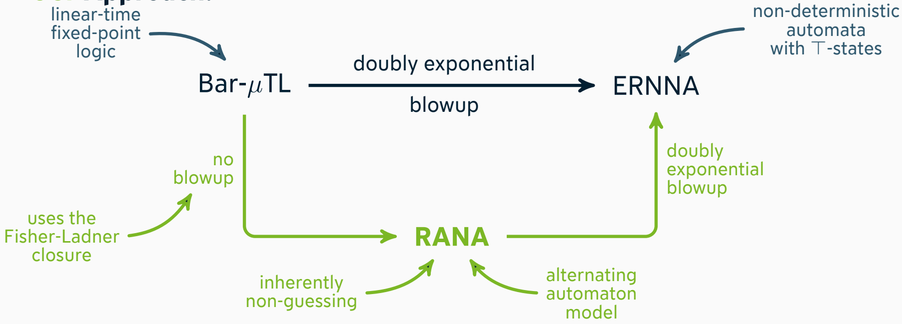

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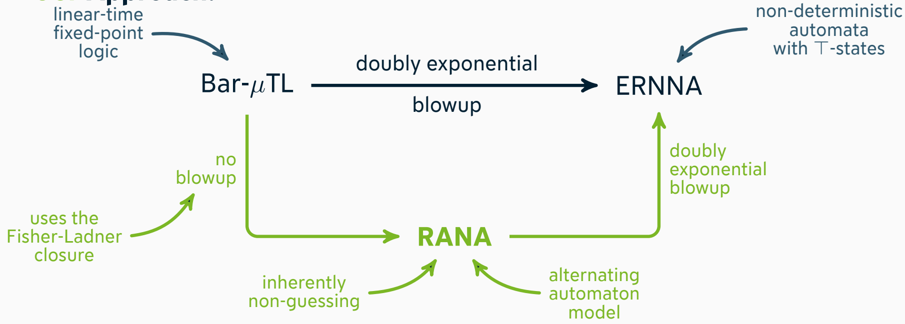

Theorem (*Expressive Equivalence*)

RANAs and Bar-μTL formulae are expressively equivalent.

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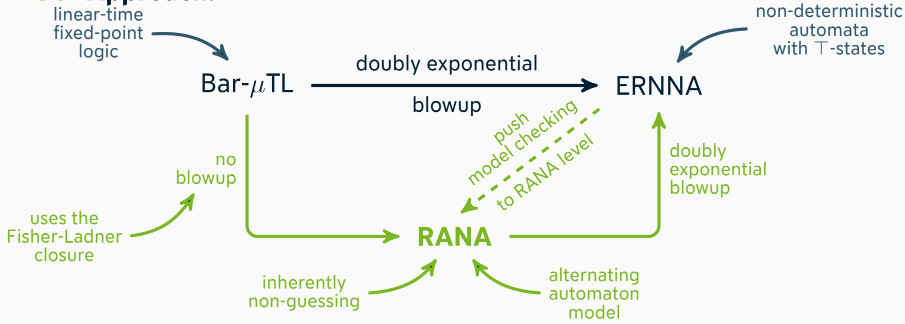

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uses the Fisher-Ladner closure → no blowup

alternating automaton model → **RANA**

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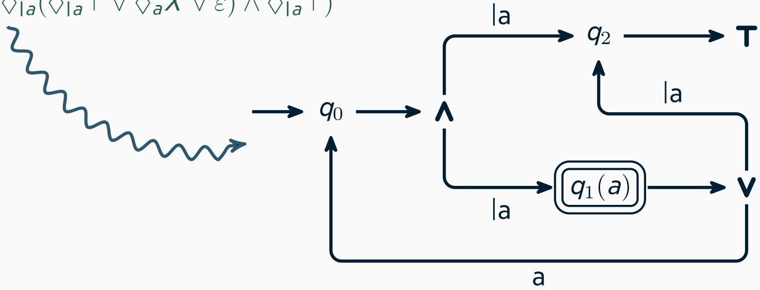

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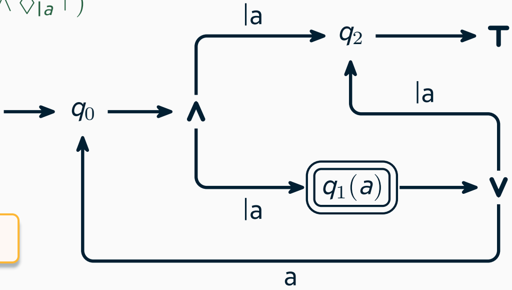

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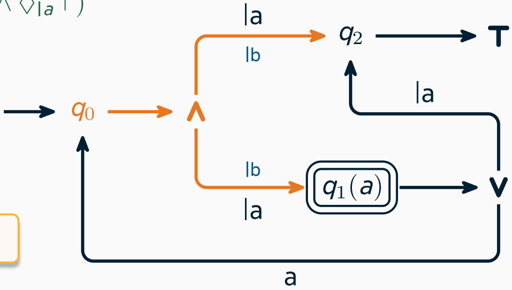

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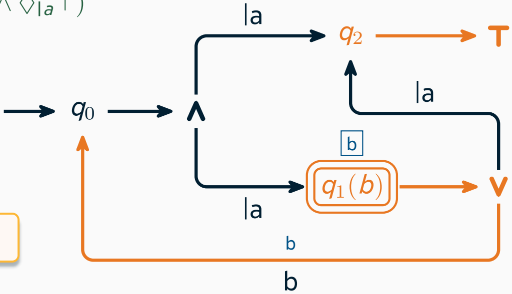

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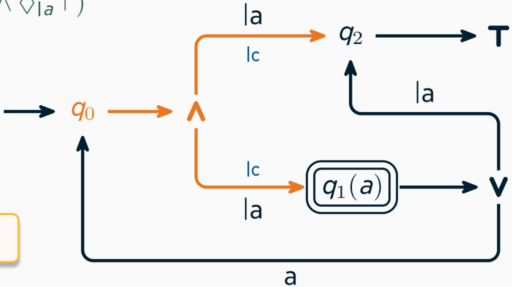

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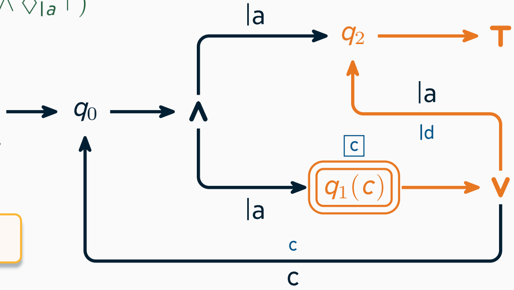

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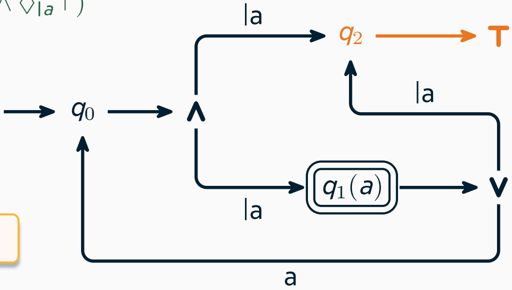

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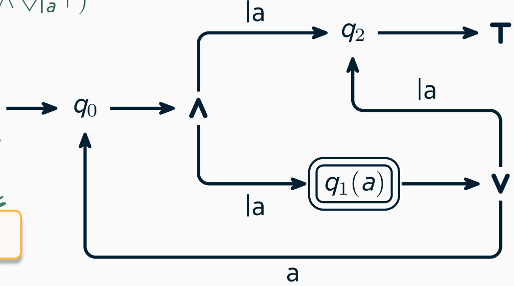

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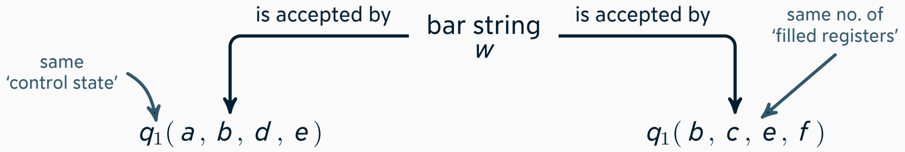

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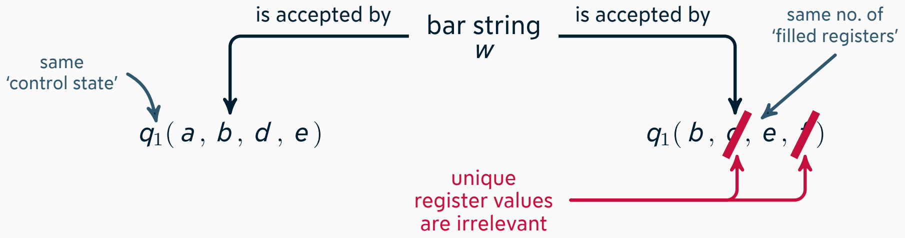

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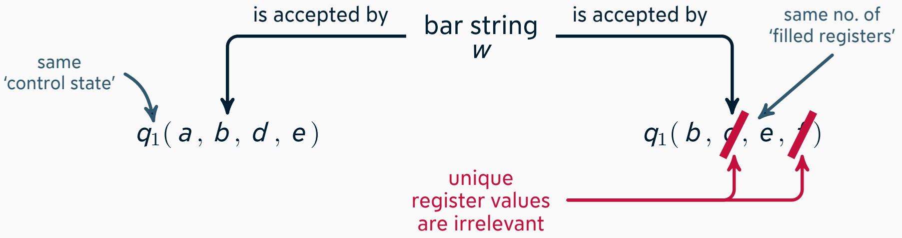

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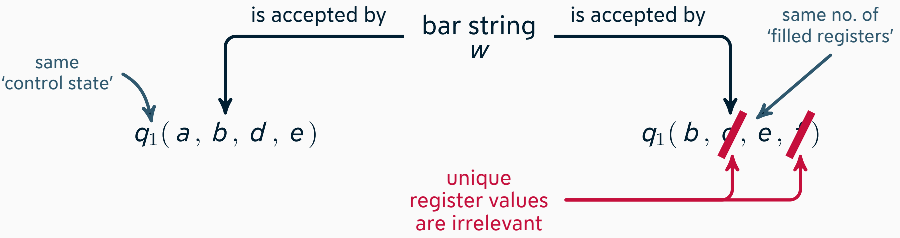

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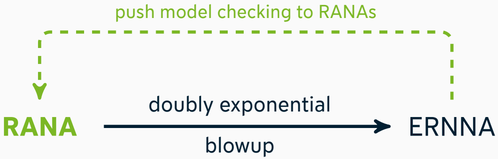
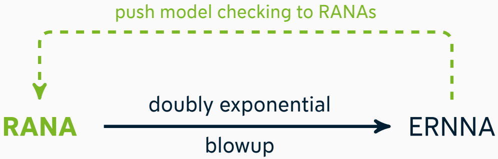
Theorem (*De-Alternation*)

RANAs can be de-alternated to ERNNAs with a doubly exponential blowup.

Finite Representability





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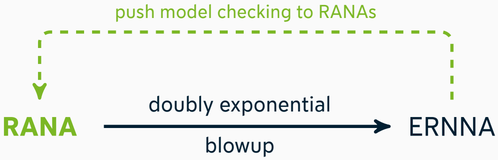

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- To push model checking back to RANAs, we saw a similar ‘equivalence’:

Theorem (***Finitisation*** **)**

Every RANA has a non-emptiness equivalent classical AFA with exponentially many states.

in the no. of ‘registers’

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RANA —exponential blowup (no. of registers)→ AFA

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  Use well-known equivalence: $L_A \subseteq L_B$ iff $L_A \cap \mathbf{comp}(L_B) = \emptyset$.
  (complement of L_B → **comp**)

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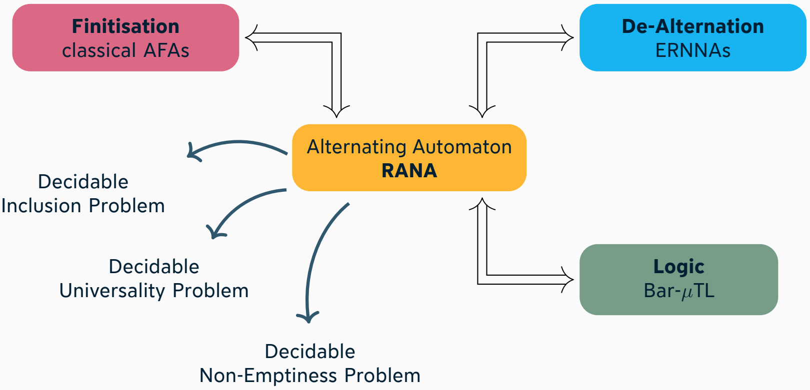
What about data languages?

For data languages, the inclusion problem is decidable in 2ExpSpace.

There is seemingly a need for de-alternation.

Conclusion & Future Work

- We looked at a variant of alternating automata for data languages with inherent name binding, and found many interesting properties:



- Some remaining problems:
  - Improve Local Freshness Complexity
  - Residuality/Learning RANAs
  - Extension to ω-Words

Questions?

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References

Hausmann, Daniel, Stefan Milius, Lutz Schröder. **'A Linear-Time Nominal μ-Calculus with Name Allocation'.** *46th International Symposium on Mathematical Foundations of Computer Science (MFCS 2021).* Ed. by Filippo Bonchi, Simon J. Puglisi. Vol. 202. LIPIcs. Dagstuhl, Germany: Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021, 58:1–58:18. ISBN: 978-3-95977-201-3. DOI: 10.4230/LIPIcs.MFCS.2021.58. URL: https://drops.dagstuhl.de/opus/volltexte/2021/14498.

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