Friedrich-Alexander-Universität
Technische Fakultät

Learning Automata with Name Allocation

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Radboud Universiteit Nijmegen

Data Languages

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» **Formal:** Closure under permutation of data values

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- **Problem** with 'unrestricted' data languages: **Unfeasability** for practical use.

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Motivation and Example

- **Restriction:** Making equality and inequality **explicit** through binders
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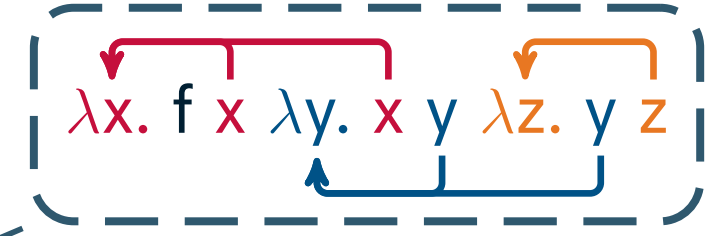

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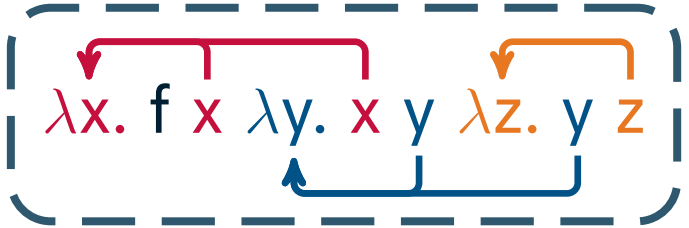

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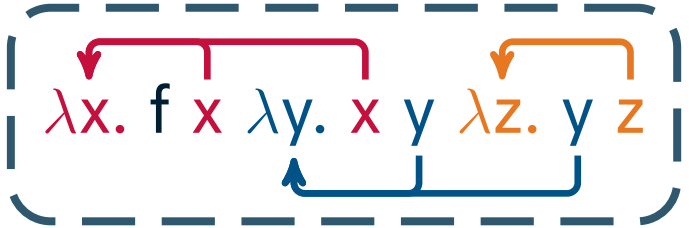

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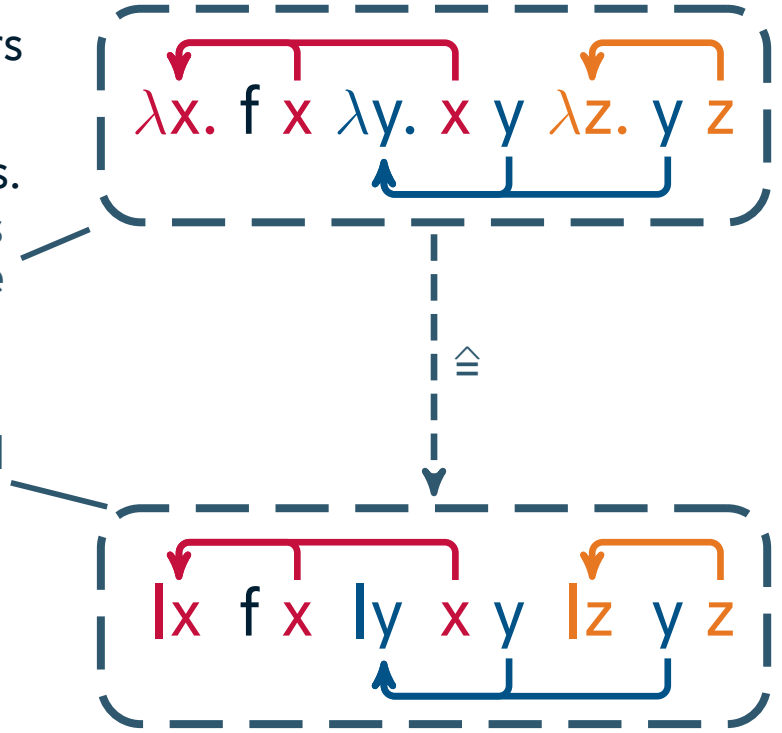

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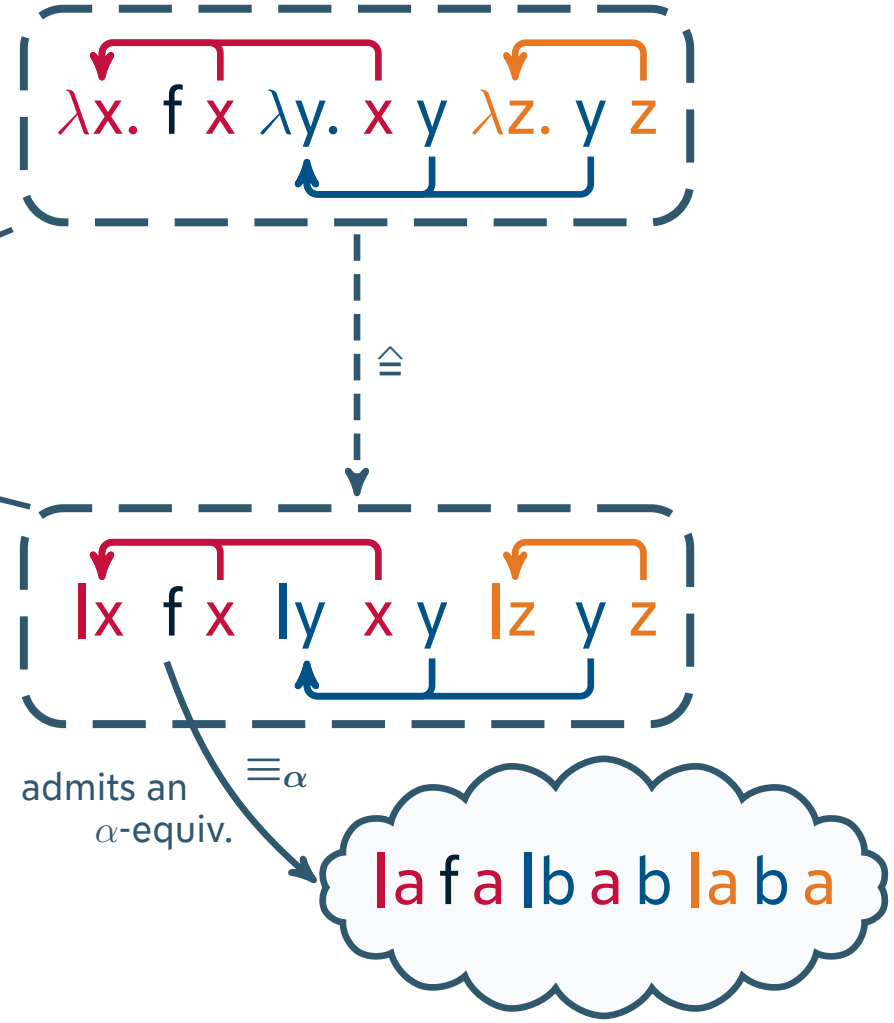

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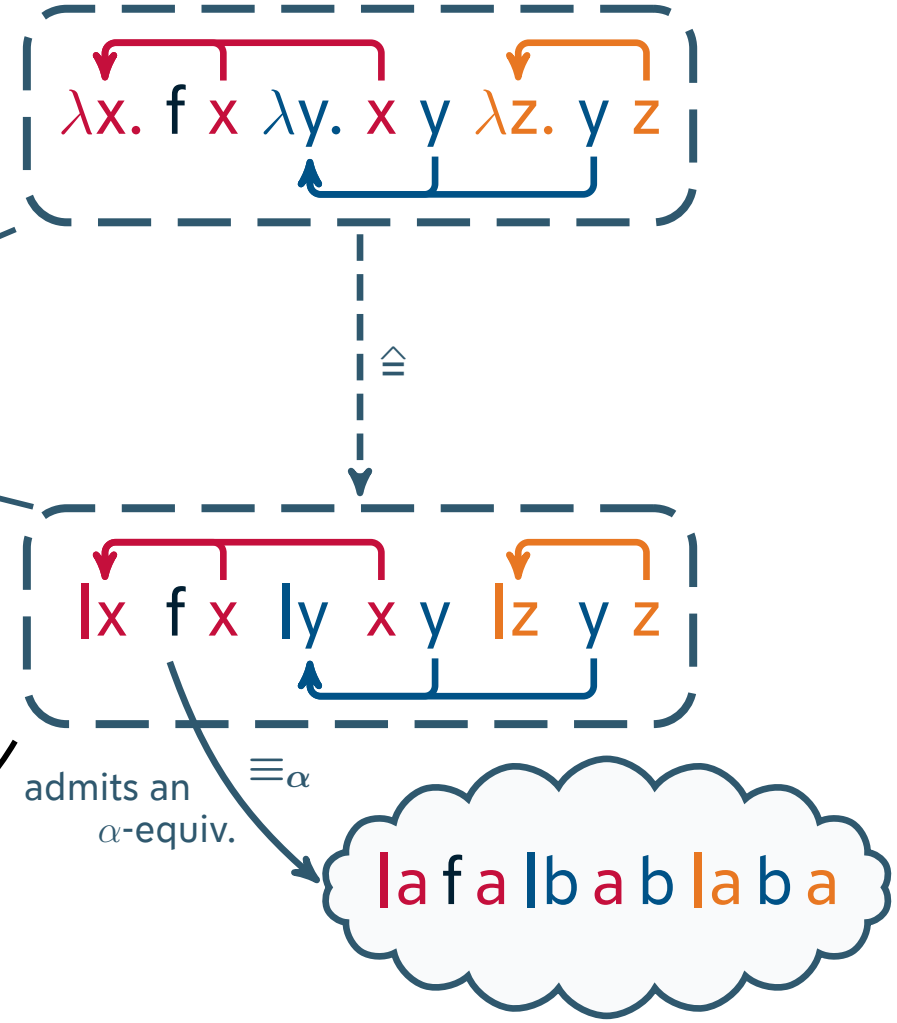

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  - **Data Languages** (over $\mathbb{D}^\star$)
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  - **Bar Languages** (closed under $\equiv_\alpha$)

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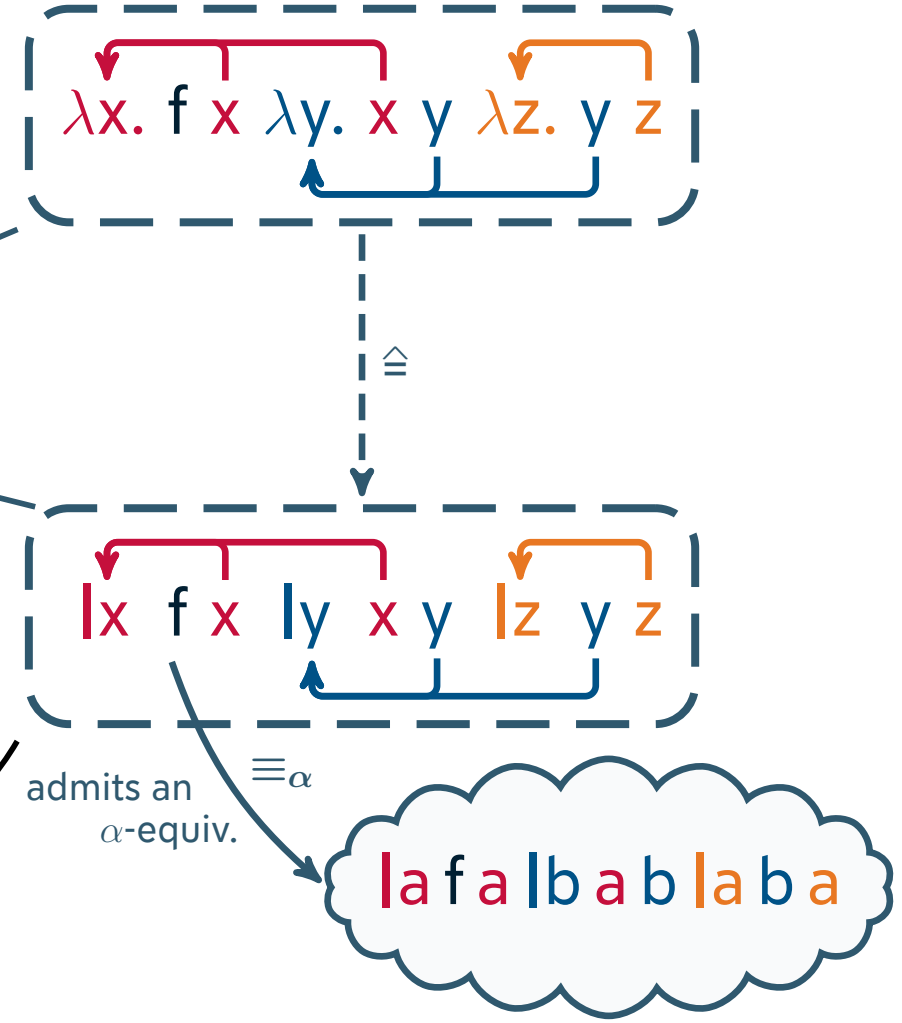

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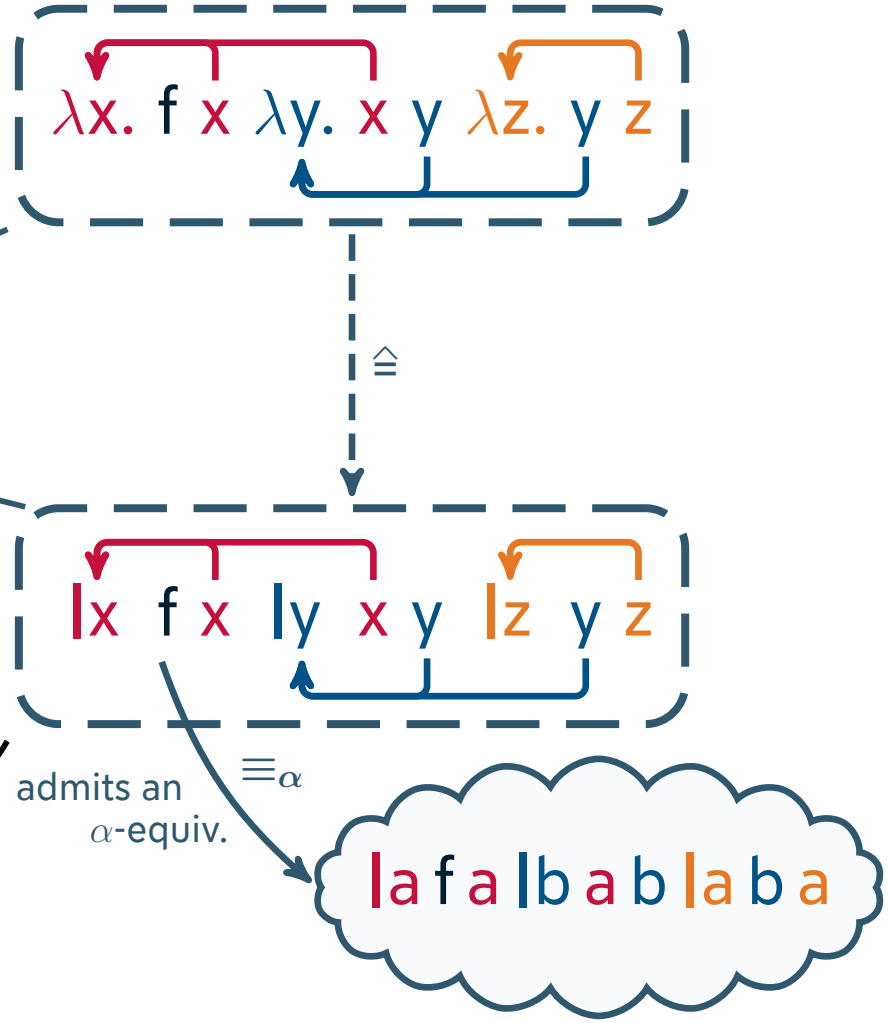

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Extensions, Automata and Acceptance

- Urbat, Hausmann, Milius, Schröder '21 introduced **infinite bar strings**. ($\overline{\mathbb{D}}^{\omega}$)
  - **α-equivalence:** all finite prefixes are α-equivalent.
  - **Acceptance:** Büchi conditions
  - **Reduction** to ultimately periodic strings possible (using quotients)

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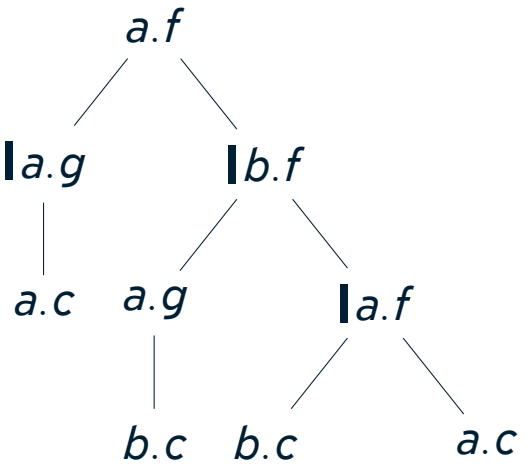

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RNTA

Acceptance via nominal Automata

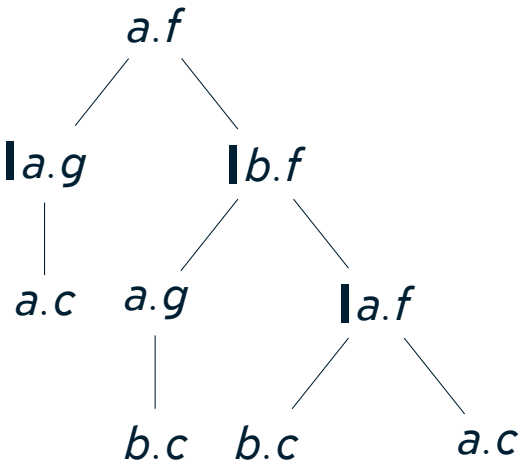
(Büchi) RNNA

DFA NFA NFTA

Acceptance via 'classical' Automata

Büchi Automata DFTA

bar automata



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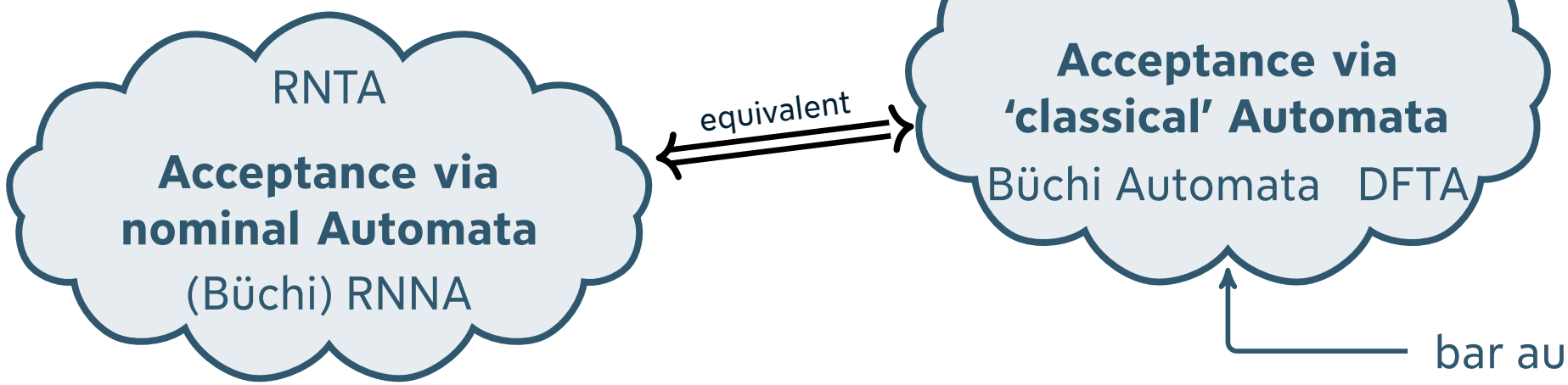

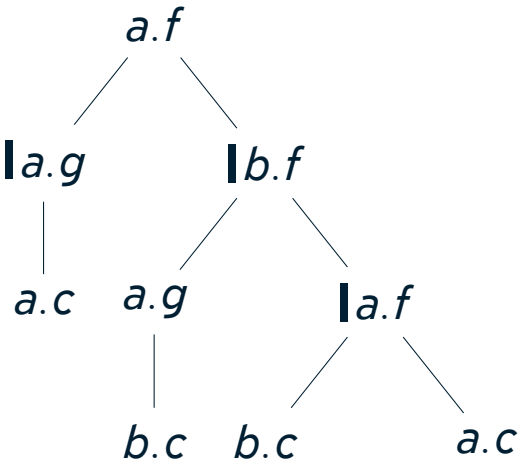

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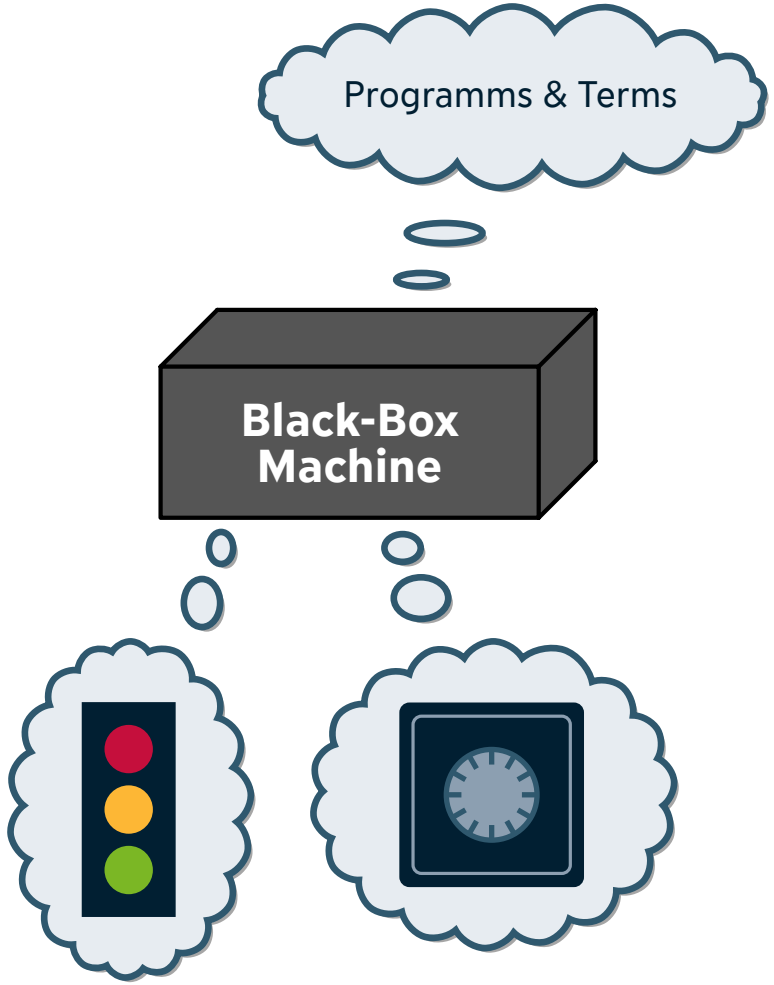

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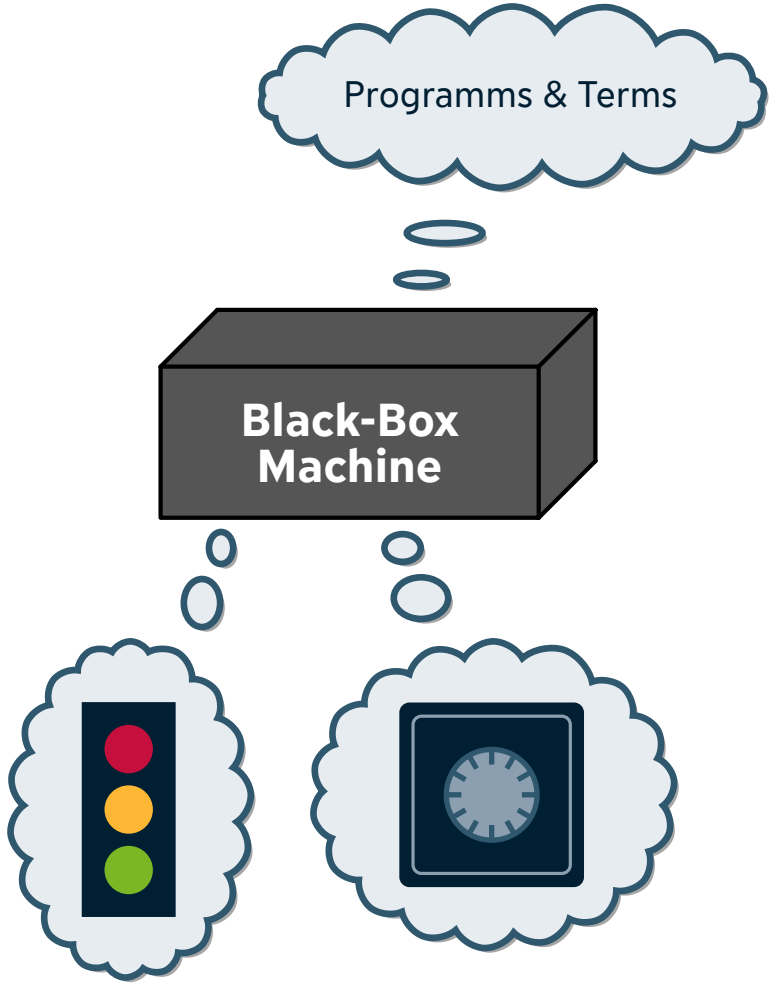

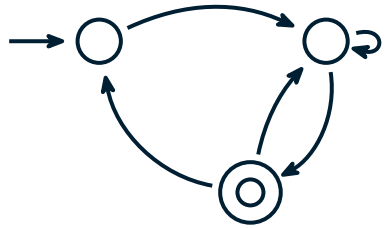

‘abstract model’

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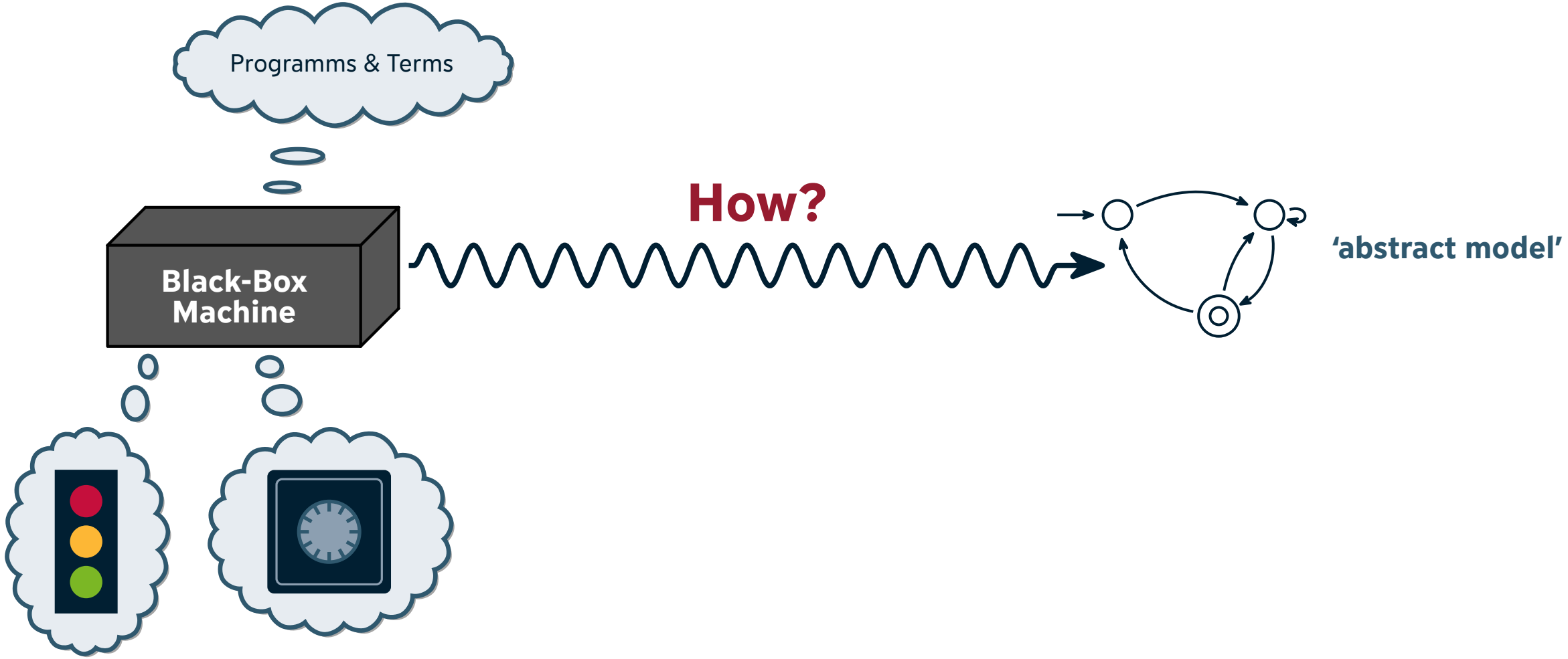

Learning Bar Automata

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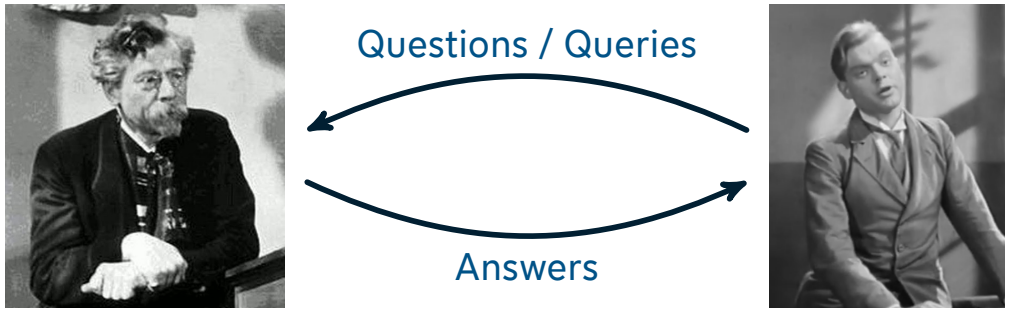

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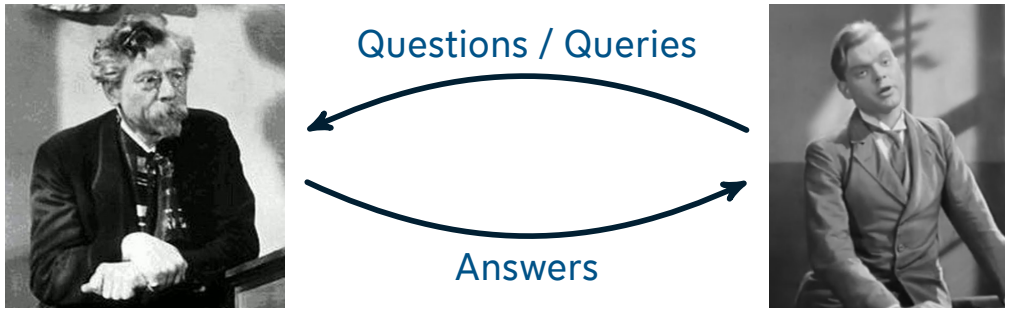

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T.CS FAU

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Learning Bar Automata

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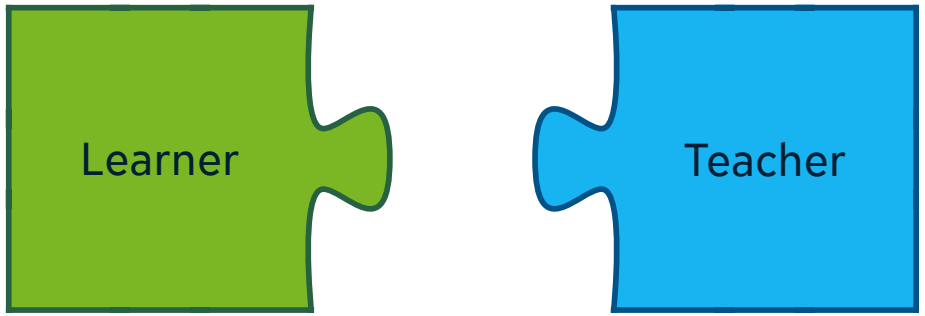

Learning Bar Automata

Example

$\overline{\mathbb{D}}_0 = \{|a, |b, b\}$

$L_T = [|a|ba]_\alpha$

Learner

Teacher

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No!

Learning Bar Automata

Example

$\overline{\mathbb{D}}_0 = \{\mathsf{l}a, \mathsf{l}b, b\}$

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Learner

Teacher

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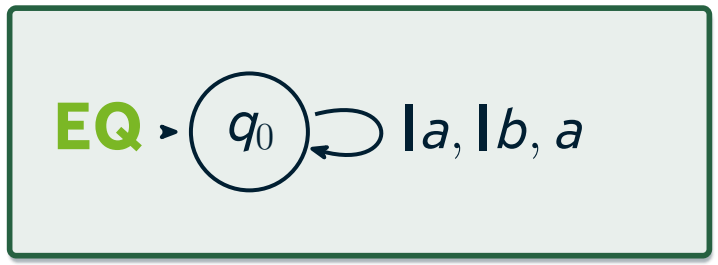

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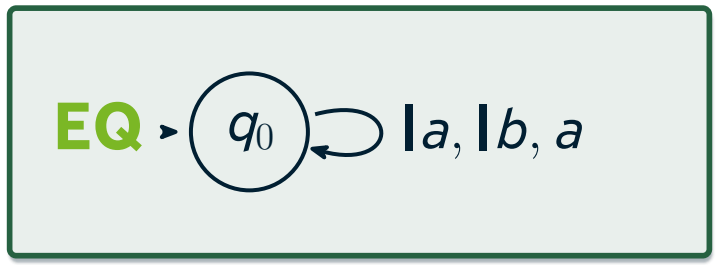

No, $w_T = |ff$ ↯

Learning Bar Automata

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Learning Bar Automata

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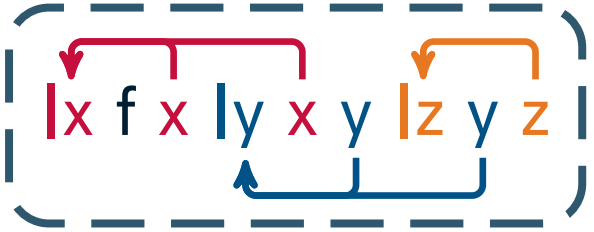

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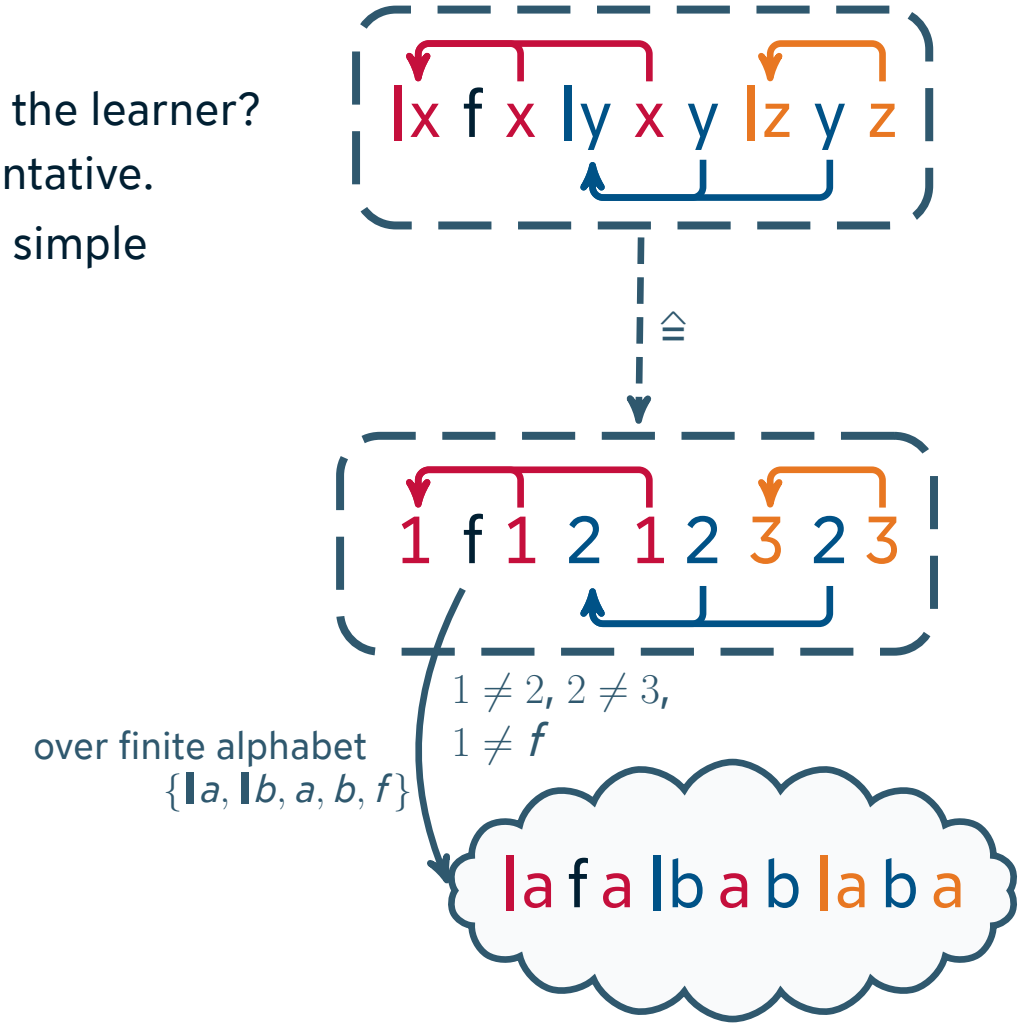

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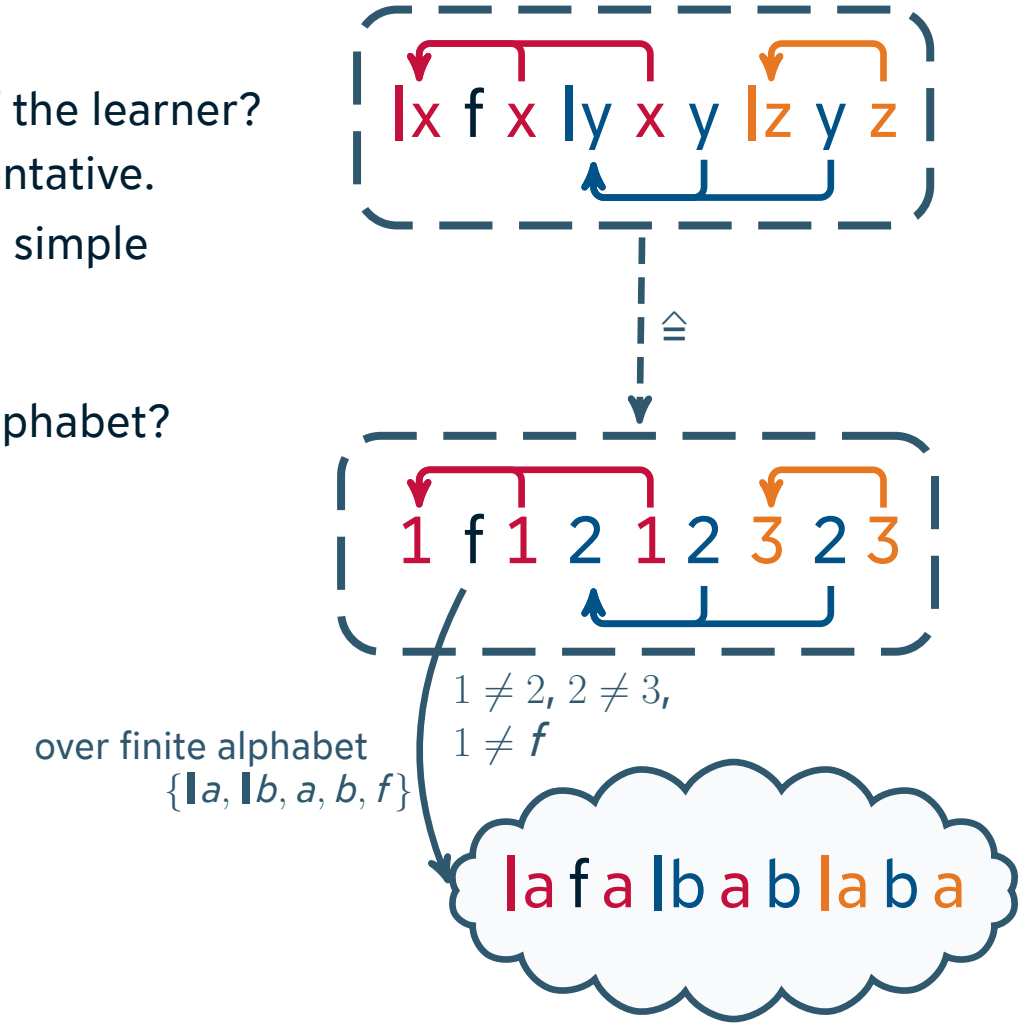

Learning Bar Automata

Example

$L_T = [\mathbf{l}a\mathbf{l}ba]_\alpha$

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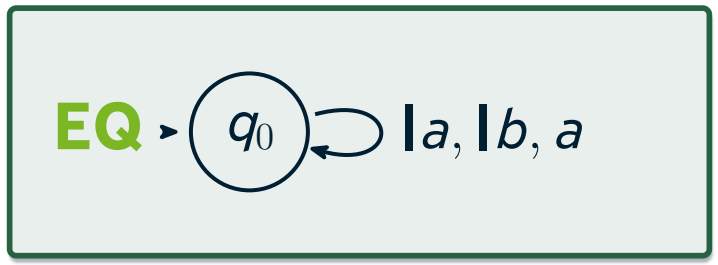
Learner

Teacher



No, $w_T = \mathbf{l}ff \longrightarrow \mathbf{l}aa$

Learning Bar Automata

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$\overline{\mathbb{D}}_0 = \{\mathsf{l}a, \mathsf{l}b, b\}$

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MQ ...

Learning Bar Automata

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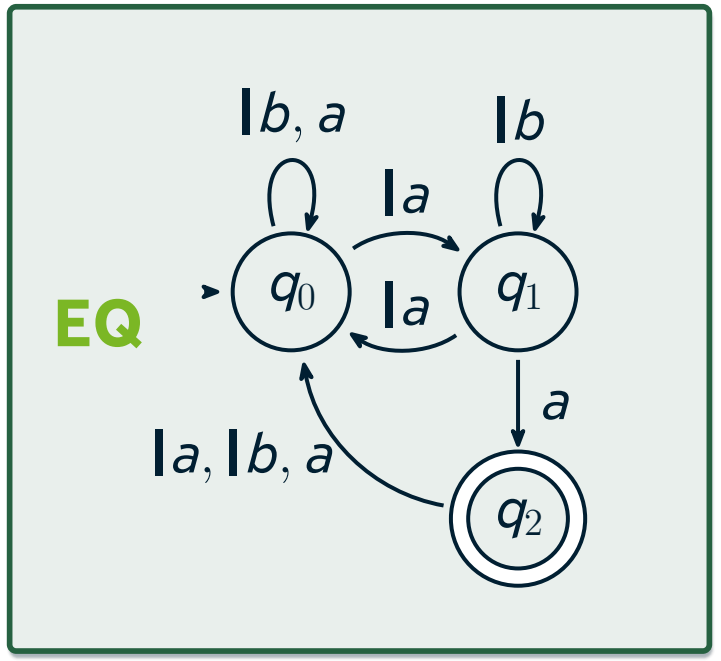
EQ



Learning Bar Automata

Example

$L_T = [\mathsf{I}a\mathsf{I}ba]_\alpha$

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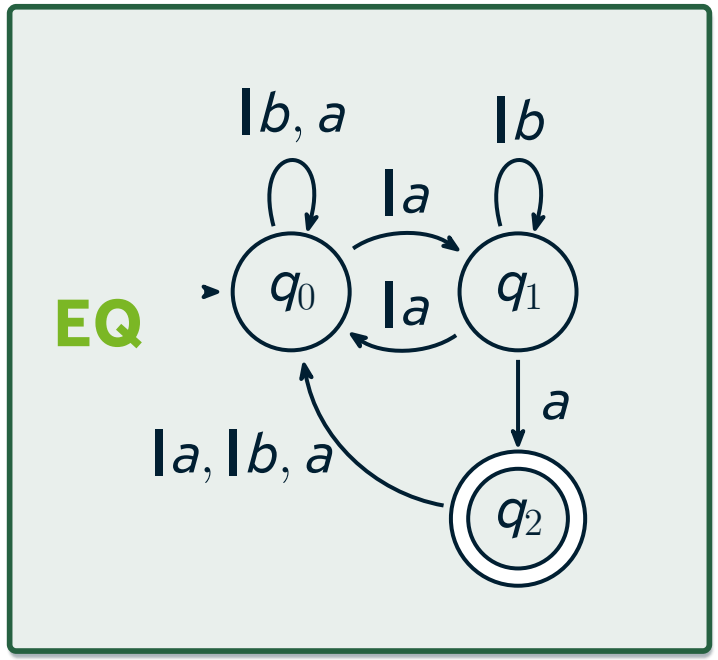
Teacher



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Learning Bar Automata

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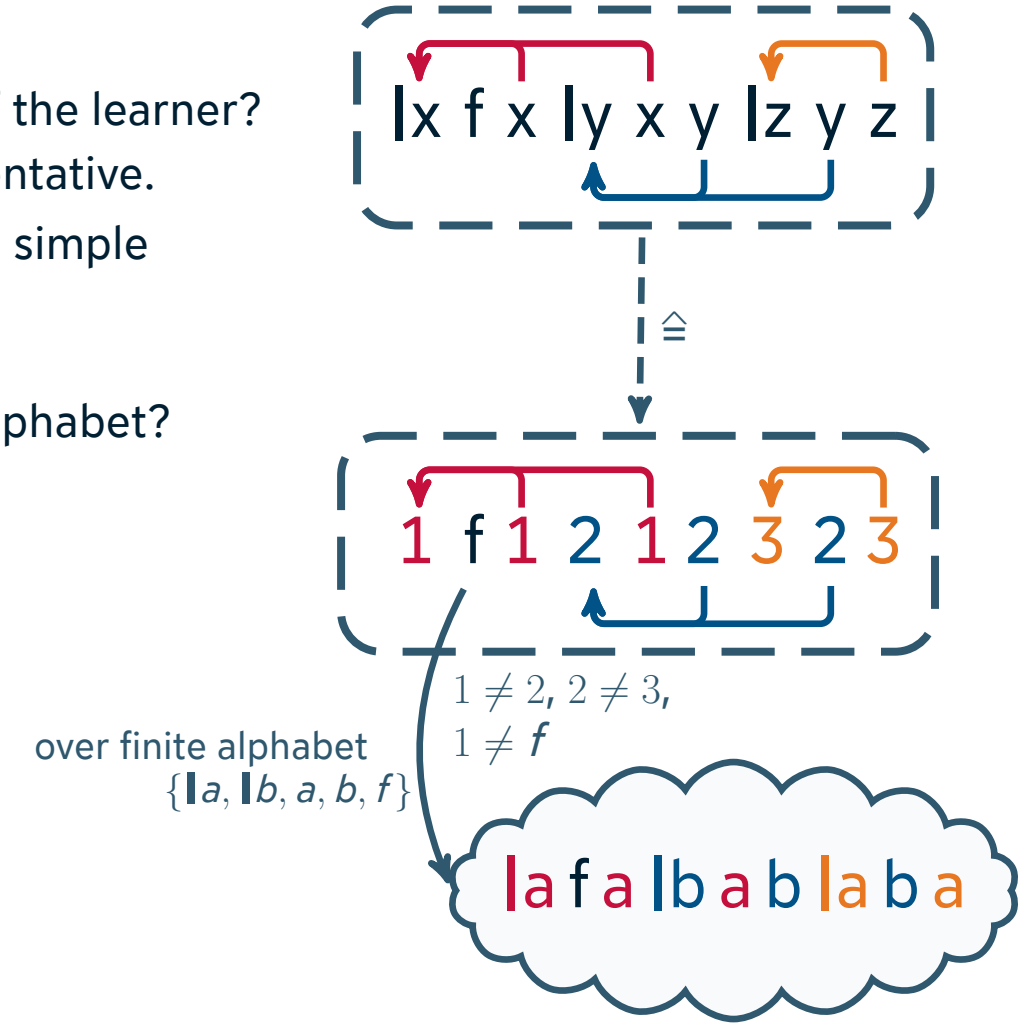

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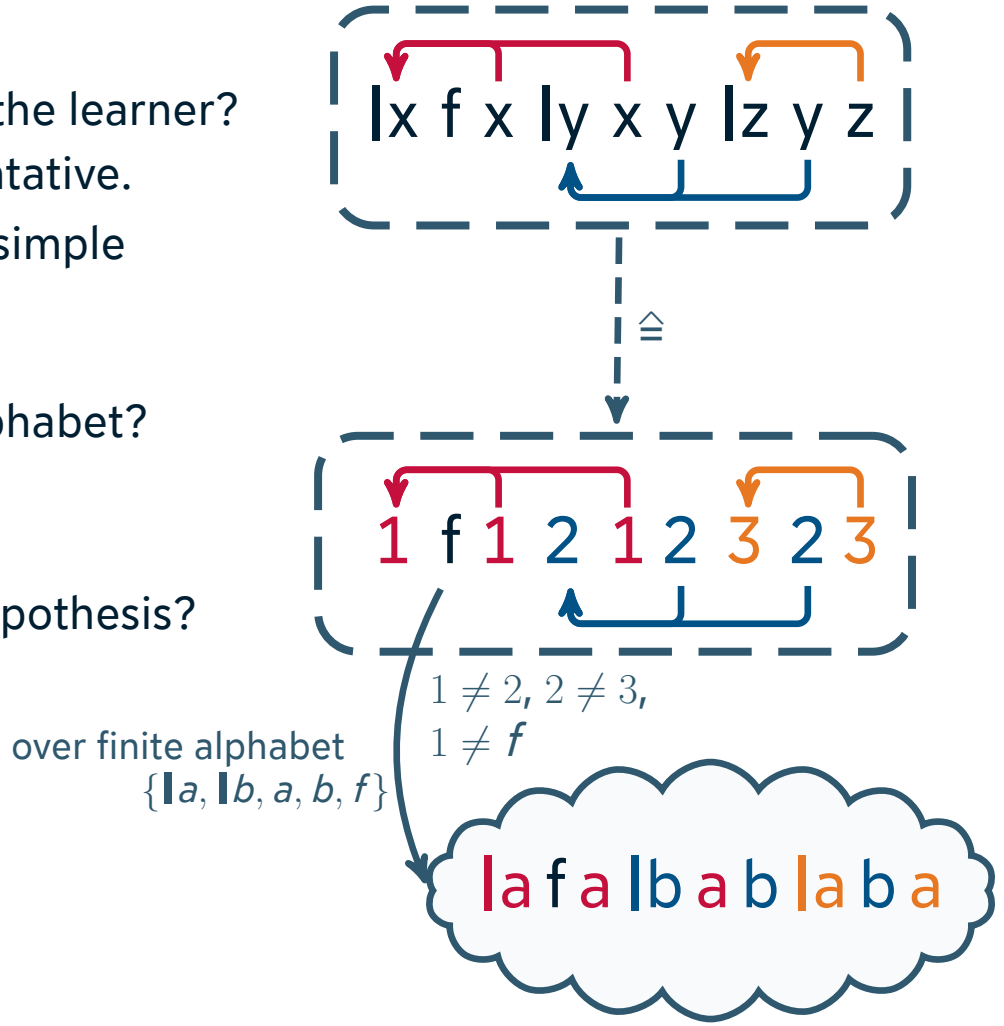

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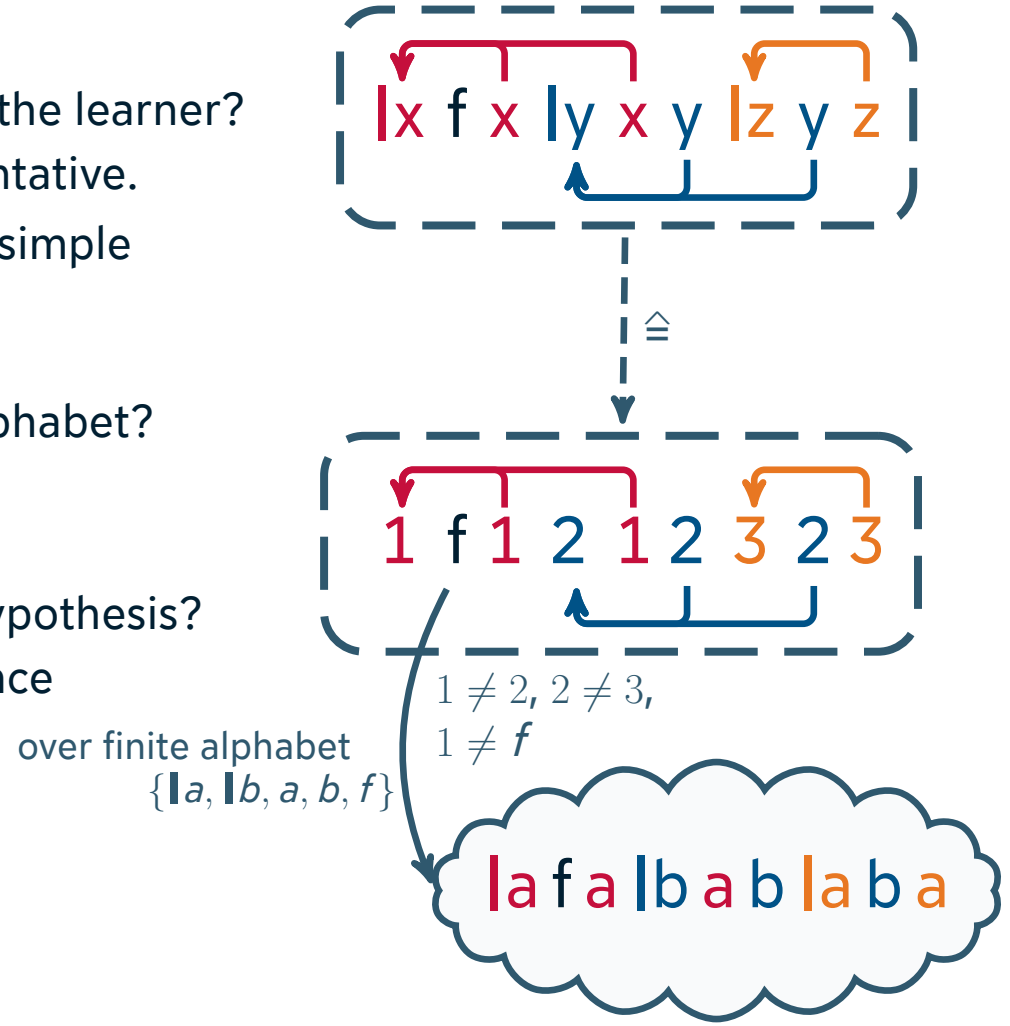
  - What if the counterexample is not directly accepted by the hypothesis?
    - We need to solve the problem of bar acceptance/acceptance modulo α-equivalence.
    - For finite bar strings, this problem is **NP-complete**.



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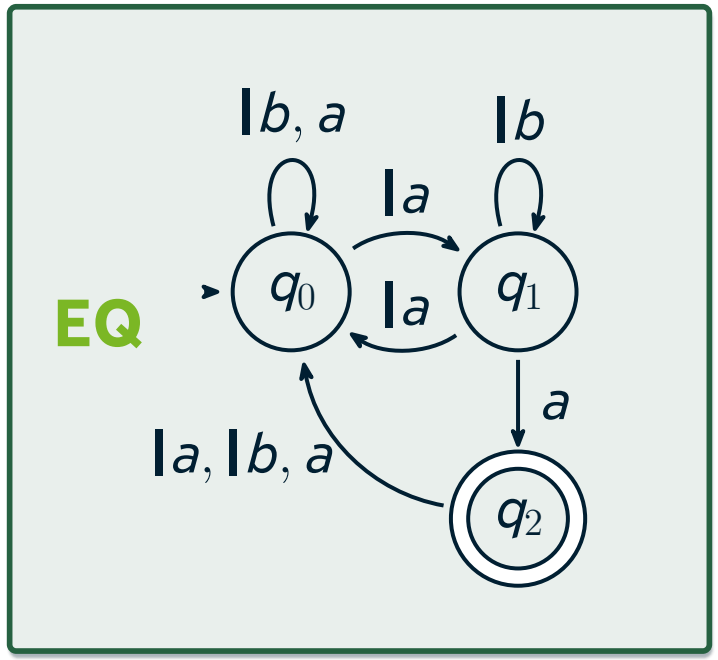

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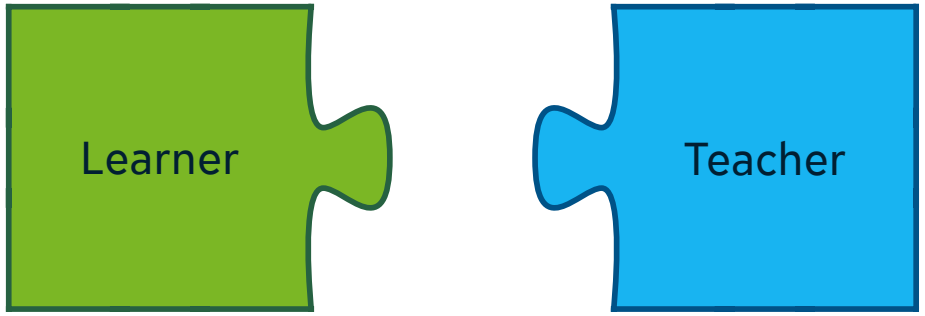

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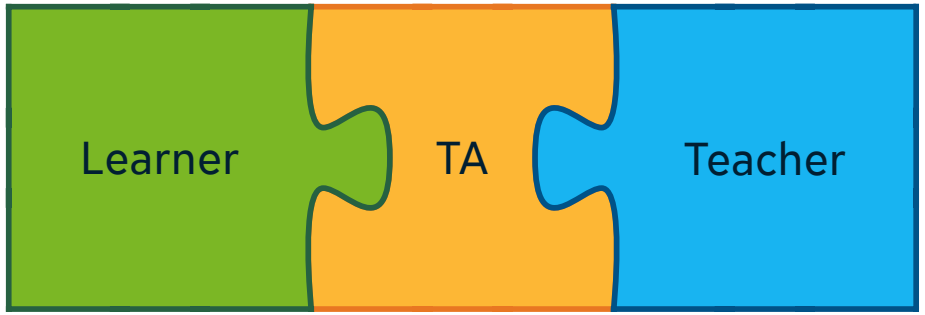

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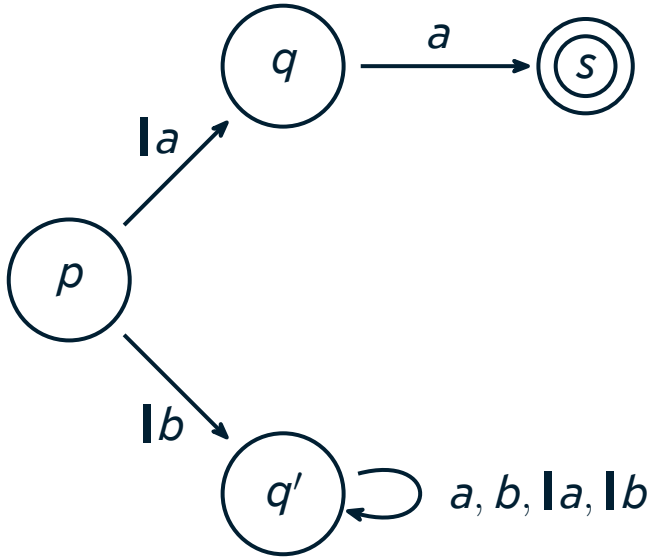

Counterexample: $|bb \in L_\alpha(\mathcal{H}) \setminus L(\mathcal{H})$ ↯

Learning Bar Automata

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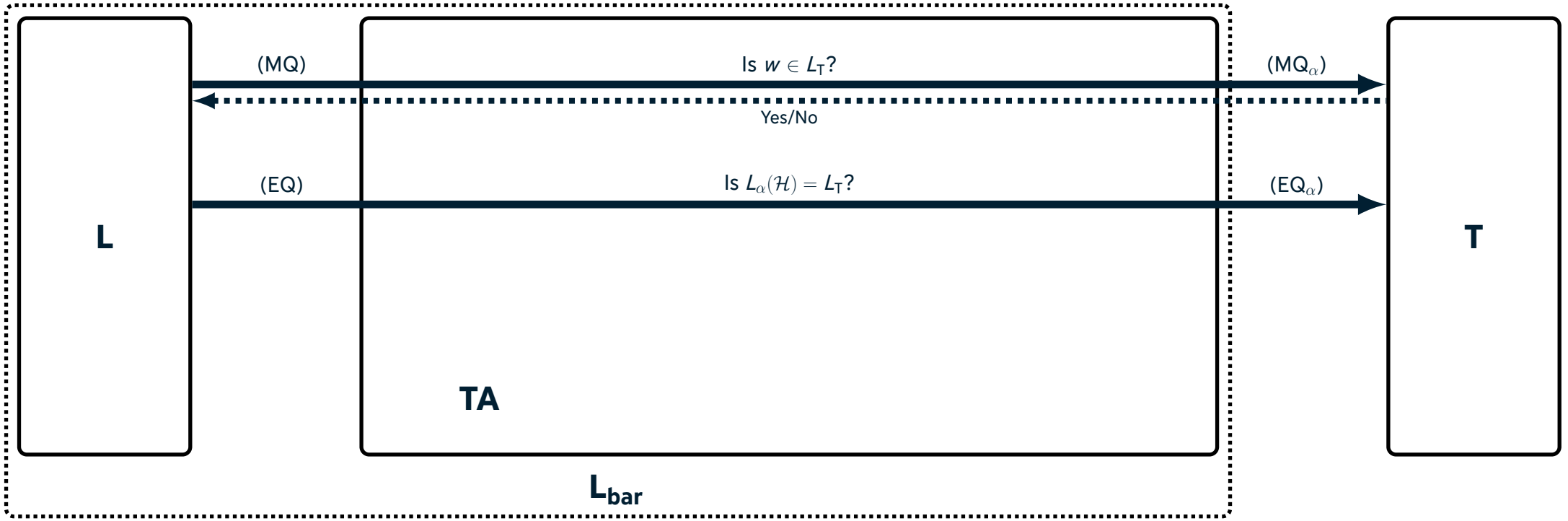

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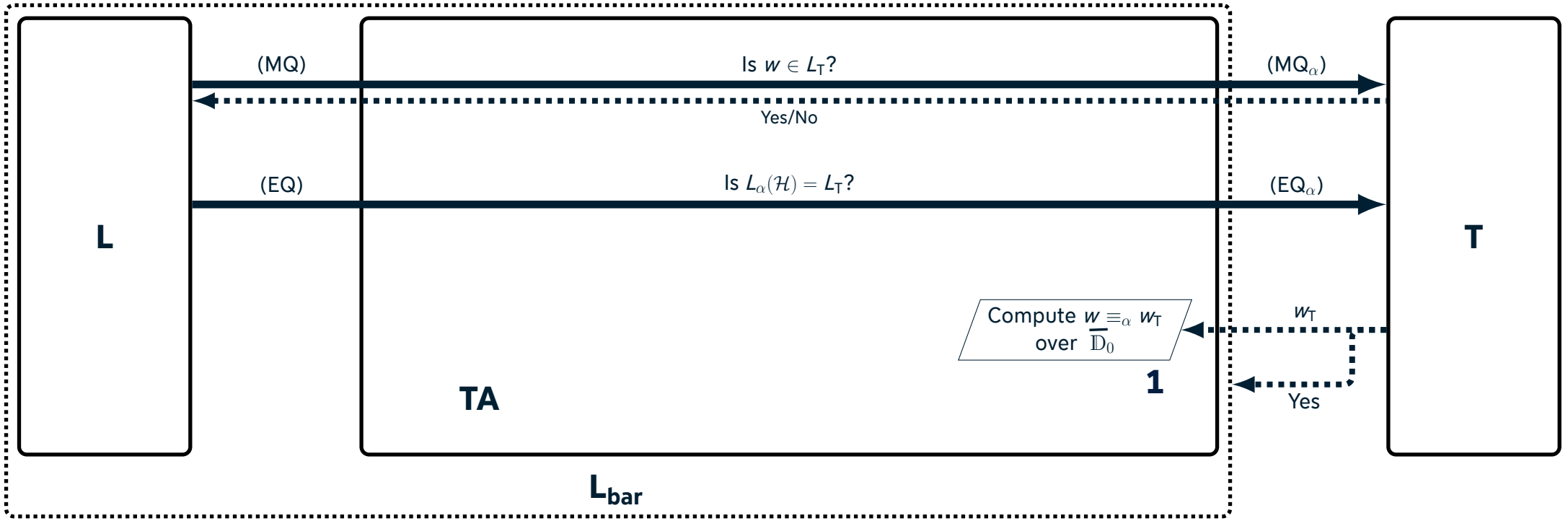

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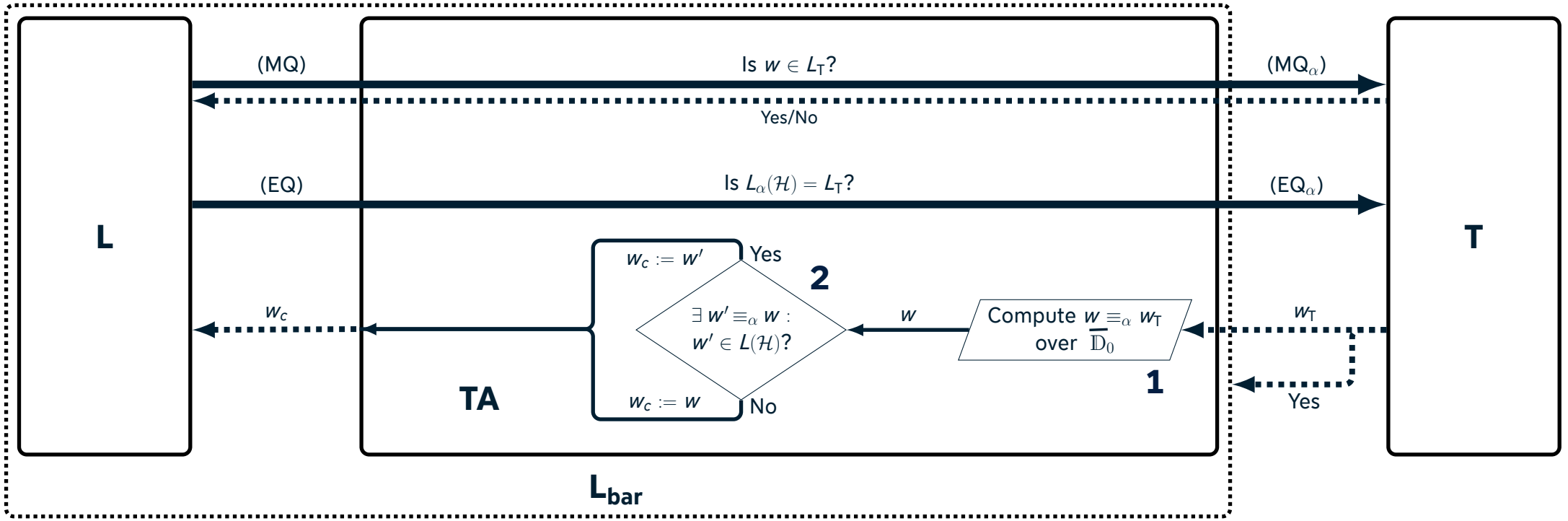

Learning Bar Automata

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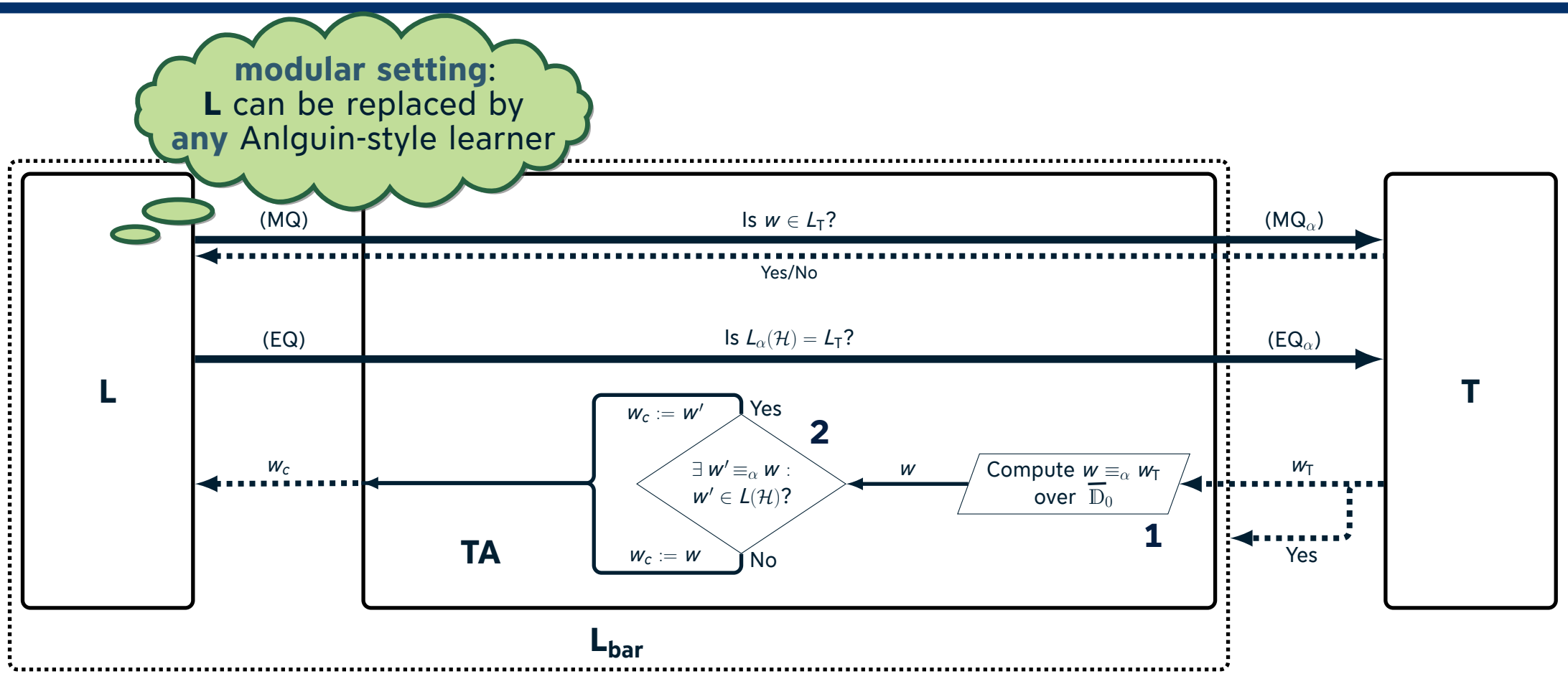

Learning Bar Languages

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No normal form? For counterexample $w_T \in \overline{\mathbb{D}}^\star$, test new $\overline{\mathbb{D}}'_\mathsf{L}$ **by cardinality**:

$$\overline{\mathbb{D}}_\mathsf{L} \subseteq \overline{\mathbb{D}}'_\mathsf{L} \subseteq (\overline{\mathbb{D}}_\mathsf{cex} \cup \overline{\mathbb{D}}_\mathsf{L})$$

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What is necessary for the arguments to go through?

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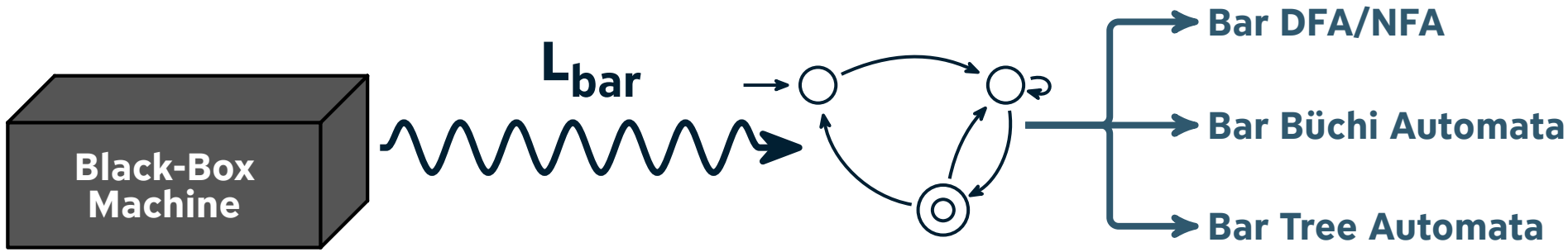
G Under these conditions, our results generalise!

Learning Bar Automata

Conclusions and Future Work

- We developed a modular learning algorithm for bar languages with a good query complexity:



- The algorithm is general in the system type under few **conditions**.

- Some remaining problems:
  - **Generalisations to other types/coalgebras**
  - **Learn data languages**
  - **Conformance Testing/Passive Learning**

References

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- Urbat, Henning, Daniel Hausmann, Stefan Milius, Lutz Schröder. **'Nominal Büchi Automata with Name Allocation'.** *32nd International Conference on Concurrency Theory (CONCUR 2021)*. Ed. by Serge Haddad, Daniele Varacca. Vol. 203. Leibniz International Proceedings in Informatics (LIPIcs). Schloss Dagstuhl – Leibniz-Zentrum für Informatik, 2021, 4:1–4:16. doi: 10.4230/LIPIcs.CONCUR.2021.4.