FMSoft

Lecture 5 — Model checking for CTL

(lecture version)

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- Why both smallest and greatest fixpoints are going to be important for us?
- Recall our goal: computing

$$[\![\phi]\!]^{\mathcal{M}} := \{s \in \mathcal{M} \mid s \vDash \phi\}$$

- There are some obvious functions $2^S \to 2^S$
- Consider $(\mathsf{EX})A := \{s \in S \mid \exists t.\, s \longrightarrow t \,\&\, t \in A\}$ ...
- ...and $(\mathsf{AX})A := \{s \in S \mid \forall t.\, s \longrightarrow t \Rightarrow t \in A\}$
- ...and $f_1(A) = [\![\phi]\!]^{\mathcal{M}} \cap (\mathsf{AX})A$ ...
- ...now contrast it with $f_2(A) = [\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{EX})A$

Equivalences for fixpoint computation

- $\mathsf{AG}\phi \equiv \phi \wedge \mathsf{AXAG}\phi$
- $\mathsf{EG}\phi \equiv \phi \wedge \mathsf{EXEG}\phi$
- $\mathsf{AF}\phi \equiv \phi \vee \mathsf{AXAF}\phi$
- $\mathsf{EF}\phi \equiv \phi \vee \mathsf{EXEF}\phi$
- $\mathsf{A}[\phi \mathsf{U} \psi] \equiv \psi \vee (\phi \wedge \mathsf{AXA}[\phi \mathsf{U} \psi])$
- $\mathsf{E}[\phi \mathsf{U} \psi] \equiv \psi \vee (\phi \wedge \mathsf{EXE}[\phi \mathsf{U} \psi])$

Denotations as fixpoins

- $[\![\mathsf{AG}\phi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cap (\mathsf{AX})[\![\mathsf{AG}\phi]\!]^{\mathcal{M}}$
- $[\![\mathsf{EG}\phi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cap (\mathsf{EX})[\![\mathsf{EG}\phi]\!]^{\mathcal{M}}$
- $[\![\mathsf{AF}\phi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{AX})[\![\mathsf{AF}\phi]\!]^{\mathcal{M}}$
- $[\![\mathsf{EF}\phi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{EX})[\![\mathsf{EF}\phi]\!]^{\mathcal{M}}$
- $[\![\mathsf{A}[\phi\,\mathsf{U}\,\psi]]\!]^{\mathcal{M}} = [\![\psi]\!]^{\mathcal{M}} \cup ([\![\phi]\!]^{\mathcal{M}} \cap (\mathsf{AX})[\![\mathsf{A}[\phi\,\mathsf{U}\,\psi]]\!]^{\mathcal{M}})$
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- ...and recall that another exercise is to verify all these equivalences hold ...

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- Is it the case that $A \subseteq [\![\mathsf{AG}\,\phi]\!]^{\mathcal{M}}$?

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- ...what do we need to show?
- Assume $A \subseteq [\![\phi]\!]^{\mathcal{M}} \cap (\mathsf{AX})A$
- Is it the case that $A \subseteq [\![\mathsf{AG}\,\phi]\!]^{\mathcal{M}}$?
- ... prove by induction that for any n, any point reachable from a given $s \in A$ in n steps lies in $A \cap [\![\phi]\!]^{\mathcal{M}}$ ...

- Note that we've now half-verified that

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- We have shown that $[\![\mathsf{AG}\,\phi]\!]^{\mathcal{M}}$ is above all postfixpoints of

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- Assume $[\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{AX})\, A \subseteq A$
- Is it the case that $[\![\mathsf{AF}\,\phi]\!]^{\mathcal{M}} \subseteq A$?

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- Recall the Knaster-Tarski Theorem ...
- ...what do we need to show?
- Assume $[\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{AX})A \subseteq A$
- Is it the case that $[\![\mathsf{AF}\phi]\!]^{\mathcal{M}} \subseteq A$?
- ...given any $s \notin A$ construct inductively $\pi \in \Pi(s)$ which does not intersect A and note it does not intersect $[\![\phi]\!]^{\mathcal{M}}$ either ...

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- ...given any $s \notin A$ construct inductively $\pi \in \Pi(s)$ which does not intersect A and note it does not intersect $[\![\phi]\!]^{\mathcal{M}}$ either ...
- Notice the different flavour of this proof

  Least fixpoints require witnessing, greatest are safety properties

- Again, we've now half-verified that

$$[\![\mathsf{AF}\,\phi]\!]^{\mathcal{M}} = [\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{AX})[\![\mathsf{AF}\,\phi]\!]^{\mathcal{M}}$$

- We have shown that $[\![\mathsf{AF}\,\phi]\!]^{\mathcal{M}}$ is below all prefixpoints of

$$f_3(A) := [\![\phi]\!]^{\mathcal{M}} \cup (\mathsf{AX})\,A$$

- An exercise for you is to show it is a prefixpoint of f_3 itself

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- ...but let us describe this "pure least fixpoints" strategy first
- We compute $[\![\phi]\!]^{\mathcal{M}}$ passing through the model and labelling states with increasingly complex subformulas of ϕ

- nothing labelled with $\perp$

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  With some care in the implementation of AF labelling, we should be able to get down to $O(|\phi| * |S| * (|S| + | \longrightarrow |))$ claimed by Huth&Ryan *Logic in Computer Science* book, § 3.6 on model-checking algorithms

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- how would they fare here?

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- More generally, a fairness condition C is any conjunction of the above three

Baier and Katoen, *Principles of Model Checking*, § 6.5

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- $\Pi_C(s)$ is the set of all C-fair paths starting at s

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- $\mathcal{M}, s \vDash \mathsf{A}_C[\phi \mathsf{U} \psi]$ if for all $\pi \in \Pi_C(s)$ there exists $n \in \mathbb{N}$ s.t. $\mathcal{M}, \pi(n) \vDash \psi$ and for any $i < n$, $\mathcal{M}, \pi(i) \vDash \phi$

Presenting FCTL

- Now let us extend the language of CTL with
- $\ldots \mathsf{E}_C \mathsf{X}\phi \mid \mathsf{E}_C[\phi \mathsf{U} \psi] \mid \mathsf{A}_C[\phi \mathsf{U} \psi]$
- $\mathcal{M}, s \vDash \mathsf{E}_C \mathsf{X}\phi$ if exists $\pi \in \Pi_C(s)$ s.t. $\mathcal{M}, \pi(1) \vDash \phi$
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- Other connectives work in a similar way

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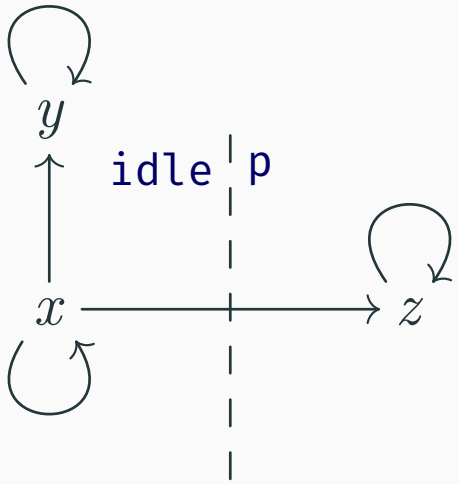
- $\mathsf{AG}(\mathsf{AGAF}\neg\mathtt{idle} \rightarrow \mathtt{p})$ does not have the same meaning ...

Example

- 

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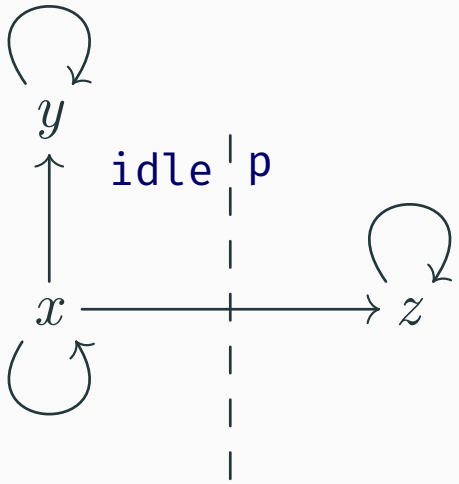
- 

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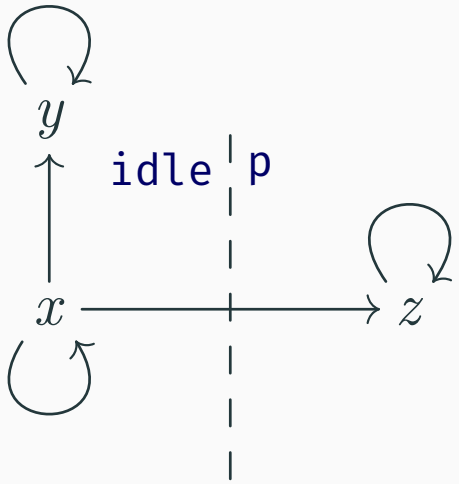
- 

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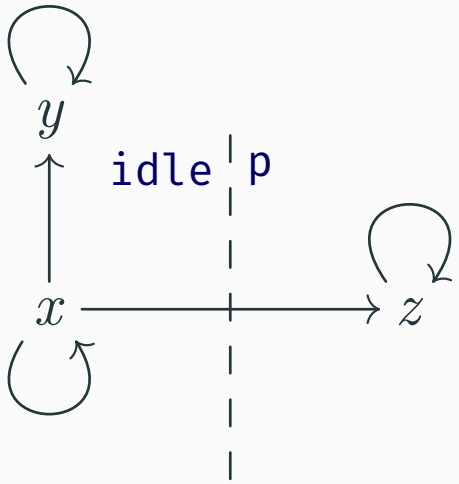
- 

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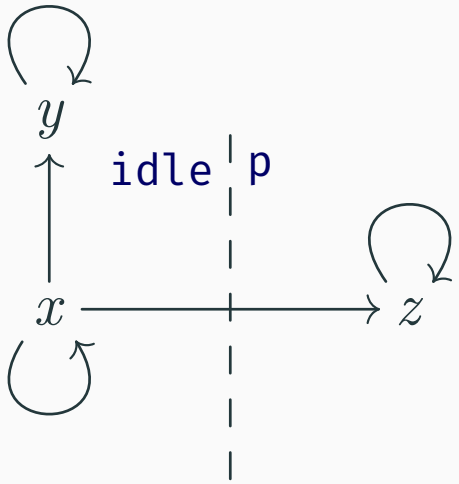
- 

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- More about comparing expressive power in the next of slides

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- We also need to pre-compute extensions of all CTL subformulas used in C

Improvement

- make sure you do, e.g., backwards breadth-first search to avoid visiting same mode
- for $[\![\mathsf{E}_C \mathsf{G} \psi]\!]^{\mathcal{M}}$, you again start with deleting non-ψ points ...
- ...find the maximal *strongly connected components (SCCs)* among those satisfying ψ ...

  our old friend Tarjan's algorithm
- and furthermore depending on C ...

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  remember that a ψ-SCC represents a ψ-suffix rather than the entire path

  Added after the lecture: Sorry for a typo in this clause

Finish as before

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- Still more powerful: fixpoint calculi and parity games

  Beyond the scope of this lecture but amazingly effective