Graded Monads

and the

Linear Time – Branching Time Spectrum

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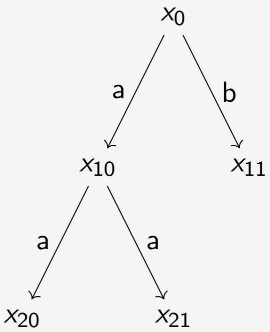
TCS Seminar SS2018

FAU Erlangen-Nürnberg

3 July 2018

Monadic Trace Semantics[1]



depth	pretraces	traces
0	$\{(\varepsilon, x_0)\}$	$\{\varepsilon\}$
1	$\{(a, x_{10}), (b, x_{11})\}$	$\{a, b\}$
2	$\{(aa, x_{20}), (aa, x_{21})\}$	$\{aa\}$
3	$\varnothing$	$\varnothing$

Embedding of $\mathcal{P}(\Sigma \times X)$ into monad $\mathcal{P}(\Sigma^* \times X)$

[1] Kurz, Milius, Pattinson, Schröder. Simplified coalgebraic trace equivalence. 2015.

Graded Monads [Smirnov]

Definition

A graded monad in $\mathcal{C}$ is a family of functors $(M_n : \mathcal{C} \to \mathcal{C})_{n \in \omega}$ together with a unit $\eta : \mathrm{Id} \to M_0$ and multiplications $\mu^{nk} : M_n M_k \to M_{n+k}$ for $n, k \in \omega$ satisfying for all $n, m, k \in \omega$:

$$\mu^{0n} \eta M_n = \mathrm{id}_{M_n} = \mu^{n0} M_n \eta$$

$$\begin{array}{ccc}
M_n M_k M_m & \xrightarrow{M_n \mu^{km}} & M_n M_{k+m} \\
\Big\downarrow{\scriptstyle \mu^{nk} M_m} & & \Big\downarrow{\scriptstyle \mu^{n,k+m}} \\
M_{n+k} M_m & \xrightarrow{\mu^{n+k,m}} & M_{n+k+m}
\end{array}$$

Generic constructions of graded monads:

- Every monad M is a graded monad $M_n = M$, $\mu^{nk} = \mu$.
- For any functor F: $M_n = F^n$, $\eta = \mathrm{id}$, $\mu^{nk} = \mathrm{id}$
- Given a monad (M, η, μ) with strength τ in a monoidal category: $M_n = M(\Sigma^n \otimes \mathrm{Id})$, $\mu^{nk} = \mu \circ M\tau$. (Example: $\mathcal{P}(\Sigma^n \times \mathrm{Id})$)
- Given a Monad (M, η, μ) and Kleisli-law $\lambda : FM \to MF$: $M_n = MF^n$, $\mu^{nk} = \mu F^{n+k} \circ M\lambda^n F^k$ with $\lambda^0 = \mathrm{id}$, $\lambda^n = \lambda^{n-1}F \circ F^{n-1}\lambda$.
- Given a monad (M, η, μ) and EM-law $\lambda : MF \to FM$: $M_n = F^n M$, $\mu^{nk} = F^{n+k}\mu \circ F^n\lambda^k M$ with $\lambda^0 = \mathrm{id}$, $\lambda^n = F\lambda^{n-1} \circ \lambda F^{n-1}$.

α-Trace Equivalence[2]

Definition

A trace semantics for G-coalgebras consists of a graded monad $((M_n), \eta, (\mu^{nk}))$ and a natural transformation

$$\alpha : G \to M_1$$

The α-pretrace $\left(\gamma^{(n)} : X \to M_n X\right)_{n \in \omega}$ for a G-coalgebra $\gamma : X \to GX$ is defined by $\gamma^{(0)} = \eta_X$ and

$$\gamma^{(n+1)} = X \xrightarrow{\alpha\gamma} M_1 X \xrightarrow{M_1 \gamma^{(n)}} M_1 M_n X \xrightarrow{\mu^{1n}} M_{n+1} X$$

The α-trace sequence is defined as

$$T_\gamma^\alpha = \left(M_n! \circ \gamma^{(n)} : X \to M_n 1\right)_{n \in \omega}.$$

In **Set** two states x, y are α-trace equivalent if $T_\gamma^\alpha(x) = T_\gamma^\alpha(y)$.

Labelled Transition Systems

Notation

Given an LTS $\gamma : X \to \mathcal{P}(\Sigma \times X)$ the set $I(p)$ for a state $p \in X$ denotes the set of outgoing labels $\pi_1[\gamma(p)]$.

$(\mathcal{P}, \eta^{\mathcal{P}}, \mu^{\mathcal{P}})$: powerset monad with strength $\tau : A \times \mathcal{P}B \to \mathcal{P}(A \times B)$

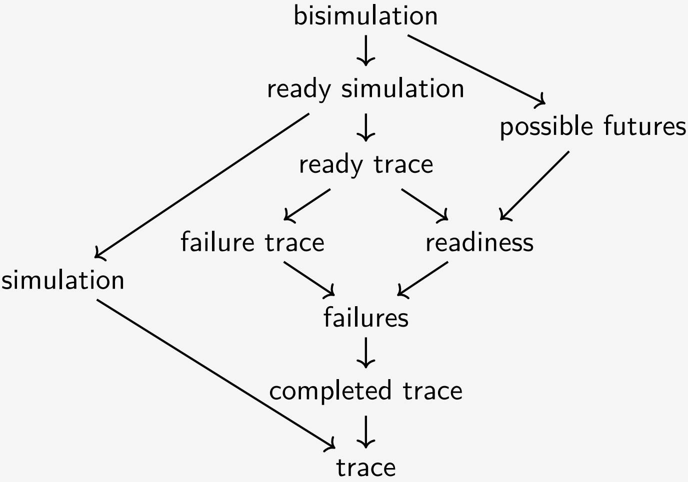
Labelled Transition Systems

Standard trace semantics for $G = \mathcal{P}(\Sigma \times \mathsf{Id})$:

$$M_n = \mathcal{P}\left(\Sigma^n \times \mathsf{Id}\right), \quad \eta(x) = \{(\varepsilon, x)\}, \quad \mu^{nk} = \mu^{\mathcal{P}} \circ \mathcal{P}\tau,$$
$$\alpha = \mathsf{id} : \mathcal{P}(\Sigma \times \mathsf{Id}) \to \mathcal{P}(\Sigma^1 \times \mathsf{Id}) = M_1$$

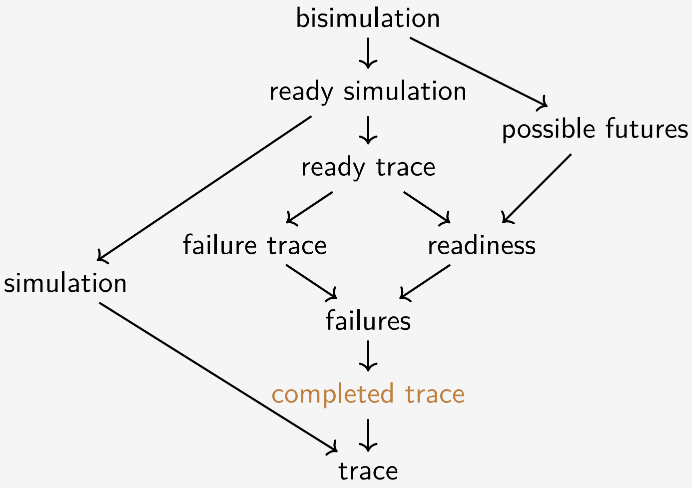
Finite depth bisimulation: $M_n = G^n$, $\eta = \mathsf{id}$, $\mu^{nk} = \mathsf{id}$, $\alpha = \mathsf{id}$

Linear Time – Branching Time Spectrum[3]



[3] van Glabeek. The Linear Time – Branching Time Spectrum I, The Semantics of Concrete, Sequential Processes. 2001.

Linear Time – Branching Time Spectrum [van Glabeek]



Completed Trace Equivalence

Definition

Two states x and y of an LTS with carrier X are completed trace equivalent if $\mathcal{CT}(x) = \mathcal{CT}(y)$ where

$$\mathcal{CT}(x) = \mathcal{T}(x) \times \left\{ w \in \Sigma^* \mid \exists z \in X \,.\, x \xrightarrow{w} z \wedge I(z) = \varnothing \right\}$$

Completed Trace Equivalence

Explicit Termination

Let $F = \Sigma \times \mathsf{Id} + 1$ and $\lambda = [\mathcal{P}\,\mathsf{inl} \circ \tau, \eta^{\mathcal{P}} \circ \mathsf{inr}] : F\mathcal{P} \to \mathcal{P}F$ a Kleisli-law. Then

$$M_n = \mathcal{P}F^n = \mathcal{P}\left(\Sigma^n \times \mathsf{Id} + \coprod_{i=0}^{n-1} \Sigma^i\right) = \mathcal{P}\left(\Sigma^n \times \mathsf{Id} + \Sigma^{<n}\right)$$

is a graded monad with $\eta(x) = \{(\varepsilon, x)\}$ and multiplication

$$\mu^{nk} = \mu^{\mathcal{P}} F^{n+k} \circ \mathcal{P}\left[\mathcal{P}\,\mathsf{inl} \circ \tau, \eta^{\mathcal{P}} \circ \mathsf{inr}\right] F^k$$

α-trace equivalence is defined by $\alpha : \mathcal{P}(\Sigma \times \mathsf{Id}) \to \mathcal{P}(\Sigma \times \mathsf{Id} + 1)$:

$$\alpha(\varnothing) = \{*\}, \quad \alpha(A) = A \text{ for } A \neq \varnothing$$

α-Trace Semantics v2

Definition

Given a family of functors $(\hat{M}_n)_n$ with associative multiplication $(\hat{\mu}^{nk})_{n,k}$ and a functor F together with natural transformations

$$\beta^n : \hat{M}_{n-1}F \to \hat{M}_n.$$

Then the following is a graded monad:

$$M_n = \hat{M}_{n-1}F, \quad M_0 = \mathrm{Id}, \quad \eta = \mathrm{id}, \quad \mu^{0n} = \mu^{n0} = \mathrm{id}$$

$$\mu^{nk} = \hat{M}_{n-1}F\hat{M}_{k-1}F \xrightarrow{\beta^n M_k} \hat{M}_n\hat{M}_{k-1}F \xrightarrow{\hat{\mu}^{n,k-1}F} \hat{M}_{n+k-1}F$$

Given $\alpha : G \to M_1 = \hat{M}_0F$ define the α-trace sequence by replacing $M_n!$ with $\hat{M}_n!$:

$$\left(\hat{M}_n!\gamma^{(n+1)} : X \to \hat{M}_n 1\right)_{n\in\omega}$$

Completed Trace Semantics

Definition

$$\hat{M}_n = \mathcal{P}\left(\Sigma^n \times \mathsf{Id} + \Sigma^n\right), \quad \hat{\mu}^{nk} = \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \mathcal{P}\pi_1 :$$
$$\mathcal{P}\left(\Sigma^n \times \mathcal{P}\left(\Sigma^k \times \mathsf{Id} + \Sigma^k\right) + \Sigma^n\right) \to \mathcal{P}\left(\Sigma^{n+k} \times \mathsf{Id} + \Sigma^{n+k}\right)$$

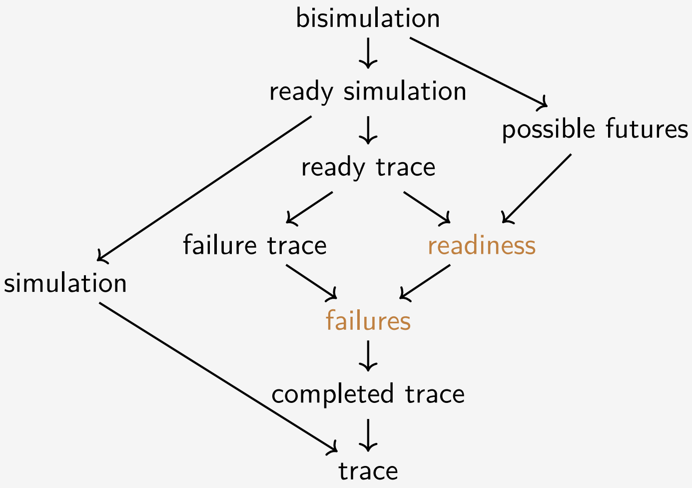
defines a graded monad $M_n = \hat{M}_{n-1}\mathcal{P}(\Sigma \times \mathsf{Id}), M_0 = \mathsf{Id}$ with

$$\beta^n = \mathcal{P}\,\mathsf{inl} \circ \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \pi_1 :$$
$$\mathcal{P}\left(\Sigma^{n-1} \times \mathcal{P}\left(\Sigma \times \mathsf{Id}\right) + \Sigma^{n-1}\right) \to \mathcal{P}\left(\Sigma^n \times \mathsf{Id} + \Sigma^n\right)$$

Completed trace equivalence is captured by α-trace equivalence for

$$\alpha : \mathcal{P}(\Sigma \times \mathsf{Id}) \to \mathcal{P}(\Sigma^0 \times \mathcal{P}(\Sigma \times \mathsf{Id}) + \Sigma^0) = M_1$$
$$\alpha(\varnothing) = \{\varepsilon\}, \quad \alpha(A) = \{(\varepsilon, A)\} \text{ for } A \neq \varnothing$$

Linear Time – Branching Time Spectrum [van Glabeek]



Failures and Readiness Semantics

Definition

Two states x and y of an LTS with carrier X are failure equivalent if $\mathcal{F}(x) = \mathcal{F}(y)$ where

$$\mathcal{F}(x) = \left\{ \langle w, A \rangle \in \Sigma^* \times \mathcal{P}\Sigma \mid \exists z \in X \,.\, x \xrightarrow{w} z \wedge A \cap I(z) = \varnothing \right\}$$

Two states x and y of an LTS with carrier X are ready equivalent if $\mathcal{R}(x) = \mathcal{R}(y)$ where

$$\mathcal{R}(x) = \left\{ \langle w, A \rangle \in \Sigma^* \times \mathcal{P}\Sigma \mid \exists z \in X \,.\, x \xrightarrow{w} z \wedge A = I(z) \right\}$$

Graded Monad for Failures and Readiness Semantics

Definition

$$\hat{M}_n = \mathcal{P}\left(\Sigma^n \times \mathrm{Id} \times \mathcal{P}\Sigma\right), \quad \hat{\mu}^{nk} = \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \mathcal{P}\pi_{12} :$$
$$\mathcal{P}\left(\Sigma^n \times \mathcal{P}\left(\Sigma^k \times \mathrm{Id} \times \mathcal{P}\Sigma\right) \times \mathcal{P}\Sigma\right) \to \mathcal{P}\left(\Sigma^{n+k} \times \mathrm{Id} \times \mathcal{P}\Sigma\right)$$

defines a graded monad with $M_n = \hat{M}_{n-1}\mathcal{P}(\Sigma \times \mathrm{Id})$, $M_0 = \mathrm{Id}$ and

$$\beta^n = \mu^{\mathcal{P}} \circ \mathcal{P}\tau : \mathcal{P}\left(\Sigma^{n-1} \times \mathcal{P}\left(\Sigma \times \mathrm{Id}\right) \times \mathcal{P}\Sigma\right) \to \mathcal{P}\left(\Sigma^n \times \mathrm{Id} \times \mathcal{P}\Sigma\right)$$

Graded Monad for Failures and Readiness Semantics

Definition

$$\hat{M}_n = \mathcal{P}\left(\Sigma^n \times \mathrm{Id} \times \mathcal{P}\Sigma\right), \quad \hat{\mu}^{nk} = \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \mathcal{P}\pi_{12} :$$

$$\mathcal{P}\left(\Sigma^n \times \mathcal{P}\left(\Sigma^k \times \mathrm{Id} \times \mathcal{P}\Sigma\right) \times \mathcal{P}\Sigma\right) \to \mathcal{P}\left(\Sigma^{n+k} \times \mathrm{Id} \times \mathcal{P}\Sigma\right)$$

defines a graded monad with $M_n = \hat{M}_{n-1}\mathcal{P}(\Sigma \times \mathrm{Id})$, $M_0 = \mathrm{Id}$ and

$$\beta^n = \mu^{\mathcal{P}} \circ \mathcal{P}\tau : \mathcal{P}\left(\Sigma^{n-1} \times \mathcal{P}\left(\Sigma \times \mathrm{Id}\right) \times \mathcal{P}\Sigma\right) \to \mathcal{P}\left(\Sigma^n \times \mathrm{Id} \times \mathcal{P}\Sigma\right)$$

Failures semantics: $\alpha : \mathcal{P}(\Sigma \times \mathrm{Id}) \to \mathcal{P}(\Sigma^0 \times \mathcal{P}(\Sigma \times \mathrm{Id}) \times \mathcal{P}\Sigma)$ with

$$\alpha(A) = \left\{(\varepsilon, A, S) \in \mathcal{P}\left(\Sigma^0 \times \mathcal{P}(\Sigma \times X) \times \mathcal{P}\Sigma\right) \mid S \cap \pi_1[A] = \varnothing\right\}$$

Graded Monad for Failures and Readiness Semantics

Definition

$$\hat{M}_n = \mathcal{P}\left(\Sigma^n \times \mathrm{Id} \times \mathcal{P}\Sigma\right), \quad \hat{\mu}^{nk} = \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \mathcal{P}\pi_{12} :$$
$$\mathcal{P}\left(\Sigma^n \times \mathcal{P}\left(\Sigma^k \times \mathrm{Id} \times \mathcal{P}\Sigma\right) \times \mathcal{P}\Sigma\right) \to \mathcal{P}\left(\Sigma^{n+k} \times \mathrm{Id} \times \mathcal{P}\Sigma\right)$$

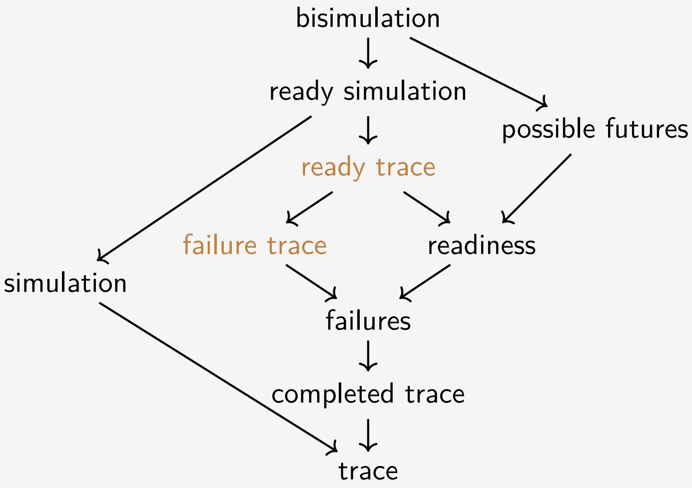
defines a graded monad with $M_n = \hat{M}_{n-1}\mathcal{P}(\Sigma \times \mathrm{Id})$, $M_0 = \mathrm{Id}$ and

$$\beta^n = \mu^{\mathcal{P}} \circ \mathcal{P}\tau : \mathcal{P}\left(\Sigma^{n-1} \times \mathcal{P}\left(\Sigma \times \mathrm{Id}\right) \times \mathcal{P}\Sigma\right) \to \mathcal{P}\left(\Sigma^n \times \mathrm{Id} \times \mathcal{P}\Sigma\right)$$

Readiness semantics: $\alpha : \mathcal{P}(\Sigma \times \mathrm{Id}) \to \mathcal{P}(\Sigma^0 \times \mathcal{P}(\Sigma \times \mathrm{Id}) \times \mathcal{P}\Sigma)$ with

$$\alpha(A) = \{(\varepsilon, A, \pi_1[A])\}$$

Linear Time – Branching Time Spectrum [van Glabeek]



Failure Trace and Ready Trace equivalence

Definition

Two states x and y of an LTS with carrier X are failure trace equivalent if $\mathcal{FT}(x) = \mathcal{FT}(y)$ where

$$\mathcal{FT}(x_0) = \Big\{ A_0 a_1 A_1 \dots a_n A_n \in (\mathcal{P}\Sigma \times \Sigma)^* \times \mathcal{P}\Sigma \mid \forall i \leq n \,.\, \exists x_i \in X \,.\, x_0 \xrightarrow{a_1} x_1 \dots \xrightarrow{a_n} x_n \wedge A_i \cap I(x_i) = \varnothing \Big\}$$

Two states x and y of an LTS with carrier X are ready trace equivalent if $\mathcal{RT}(x) = \mathcal{RT}(y)$ where

$$\mathcal{RT}(x_0) = \Big\{ A_0 a_1 A_1 \dots a_n A_n \in (\mathcal{P}\Sigma \times \Sigma)^* \times \mathcal{P}\Sigma \mid \forall i \leq n \,.\, \exists x_i \in X \,.\, x_0 \xrightarrow{a_1} x_1 \dots \xrightarrow{a_n} x_n \wedge A_i = I(x_i) \Big\}$$

Graded Monad for Failure Trace/Ready Trace Semantics

Definition

$$\hat{M}_n = \mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^n \times \mathsf{Id} \times \mathcal{P}\Sigma\right), \quad \mu^{nk} = \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \mathcal{P}\pi_{12} :$$
$$\mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^n \times \mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^k \times \mathsf{Id} \times \mathcal{P}\Sigma\right) \times \mathcal{P}\Sigma\right) \to$$
$$\mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^{n+k} \times \mathsf{Id} \times \mathcal{P}\Sigma\right)$$

defines a graded monad $M_n = \hat{M}_{n-1}\mathcal{P}(\Sigma \times \mathsf{Id})$, $M_0 = \mathsf{Id}$ with

$$\beta^n = \mathcal{P}(\langle \mathsf{id}, \varnothing ! \rangle) \circ \mu^{\mathcal{P}} \circ \mathcal{P}\tau :$$
$$\mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^{n-1} \times \mathcal{P}(\Sigma \times \mathsf{Id}) \times \mathcal{P}\Sigma\right) \to$$
$$\mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^n \times \mathsf{Id} \times \mathcal{P}\Sigma\right)$$

Graded Monad for Failure Trace/Ready Trace Semantics

$$M_n = \hat{M}_{n-1}\mathcal{P}(\Sigma \times \mathsf{Id}) = \mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^{n-1} \times \mathcal{P}(\Sigma \times \mathsf{Id}) \times \mathcal{P}\Sigma\right)$$

$$\alpha : \mathcal{P}(\Sigma \times \mathsf{Id}) \to \mathcal{P}\left((\mathcal{P}\Sigma \times \Sigma)^{0} \times \mathcal{P}(\Sigma \times \mathsf{Id}) \times \mathcal{P}\Sigma\right)$$

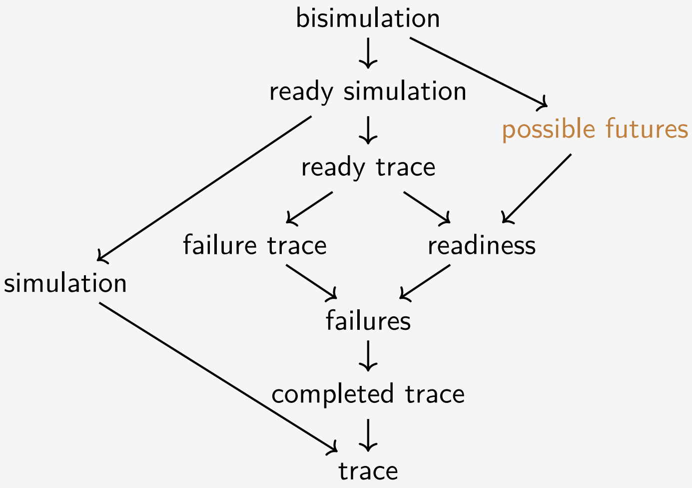
Failure Trace Semantics:

$$\alpha(A) = \{(*, A, S) \in \mathcal{P}\,(1 \times \mathcal{P}(\Sigma \times X) \times \mathcal{P}\Sigma) \mid S \cap \pi_1[A] = \varnothing\}$$

Ready Trace Semantics:

$$\alpha(A) = \{(*, A, \pi_1[A])\}$$

Linear Time – Branching Time Spectrum [van Glabeek]



Possible Futures Semantics

Definition

Two states x, y of an LTS with carrier X are possible futures equivalent if $\mathcal{PF}(x) = \mathcal{PF}(y)$ where

$$\mathcal{PF}(x) = \left\{ \langle w, T \rangle \in \Sigma^* \times \mathcal{P}\left(\Sigma^*\right) \mid \exists z \in X \, . \, x \xrightarrow{w} z \wedge T = \mathcal{T}(x) \right\}$$

Graded Monad for Possible Futures Semantics

$$M_n = \mathcal{P}\left(\coprod_{j\leq n} \Sigma^j \times \mathcal{P}(\Sigma^{n-j} \times \mathsf{Id})\right)$$

$$\mu^{nk} = \iota^{nk} \circ \langle \mu^{\mathcal{P}} \circ \mathcal{P}\tau \circ \mu^{\mathcal{P}} \circ \mathcal{P}f_n, \mathcal{P}(\coprod_{j\leq n} \Sigma^j \times \mu^{\mathcal{P}}\mathcal{P}\tau) \circ M_n\mu^{\mathcal{P}} \circ M_n\mathcal{P}f_k\rangle$$

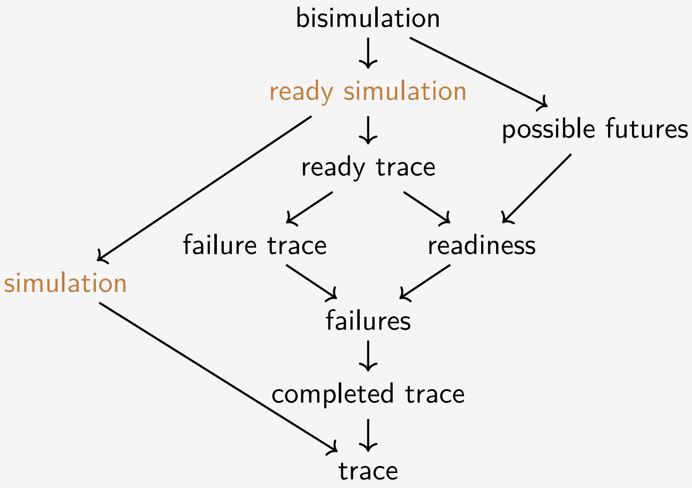
$$f_n = \mathcal{P}[\mathsf{id}]_{j\leq n} \circ \coprod_{j\leq n} \tau : \coprod_{j\leq n} \Sigma^j \times \mathcal{P}(\Sigma^{n-j} \times \mathsf{Id}) \to \mathcal{P}(\Sigma^n \times \mathsf{Id})$$

$$\iota^{nk} : \mathcal{P}\left(\Sigma^n \times \coprod_{i\leq k} \Sigma^i \times \mathcal{P}(\Sigma^{k-i} \times \mathsf{Id})\right) \times \mathcal{P}\left(\coprod_{j\leq n} \Sigma^j \times \mathcal{P}(\Sigma^{n+k-j} \times \mathsf{id})\right)$$

$$\to \mathcal{P}\left(\coprod_{n\leq j\leq n+k} \Sigma^j \times \mathcal{P}(\Sigma^{n+k-j} \times \mathsf{Id}) + \coprod_{j\leq n} \Sigma^j \times \mathcal{P}(\Sigma^{n+k-j} \times \mathsf{Id})\right)$$

$$\to \mathcal{P}\left(\coprod_{j\leq n+k} \Sigma^j \times \mathcal{P}(\Sigma^{n+k-j} \times \mathsf{Id})\right)$$

Linear Time – Branching Time Spectrum [van Glabeek]



(Ready) Simulation Semantics

Definition

Given two LTS with carriers X and Y a simulation is a relation $R \subseteq X \times Y$ such that

$$\forall a \in \Sigma \,.\, xRy \wedge x \xrightarrow{a} x' \Rightarrow \exists y' \in Y \,.\, y \xrightarrow{a} y' \wedge x'Ry'$$

x can be simulated by y if there is a simulation R with xRy; x and y are simulation equivalent if there are simulations R, S s.t. xRy and ySx.

Definition

A ready simulation R is a simulation that additionally requires

$$xRy \Rightarrow I(x) = I(y)$$

Simulation Semantics

Definition

The set $\mathcal{L}_S$ of simulation formulas over Σ is defined as follows:

- Given an index set V and $\phi_i \in \mathcal{L}_S$ then $\bigwedge_{i \in V} \phi_i \in \mathcal{L}_S$.
- If $\phi \in \mathcal{L}_S$ then $a\phi \in \mathcal{L}_S$ for $a \in \Sigma$.

Satisfaction of simulation formulas is defined as

- $p \models \bigwedge_{i \in V} \phi_i$ if $p \models \phi_i$ for all $i \in V$.
- $p \models a\phi$ if $p \xrightarrow{a} q$ and $q \models \phi$.

Two states are simulation equivalent if they satisfy the same (infinitary) simulation formulas.

$$M_n = F^n, \quad \eta = \mathrm{id}, \quad \mu^{nk} = \mathrm{id}$$

Simulation: $F = \mathcal{P}\mathcal{P}(\Sigma \times \mathrm{Id})$, $\alpha(A) = \mathcal{P}(A)$.

Ready Simulation Semantics

Definition

The set $\mathcal{L}_S$ of ready simulation formulas over Σ is defined as

- Given an index set V and $A\phi_i \in \mathcal{L}_S$ then $A \bigwedge_{i\in V} \phi_i \in \mathcal{L}_S$.
- If $\phi \in \mathcal{L}_S$ then $Aa\phi \in \mathcal{L}_S$ for $a \in \Sigma$, $A \subseteq \Sigma$.

Satisfaction of simulation formulas is defined as

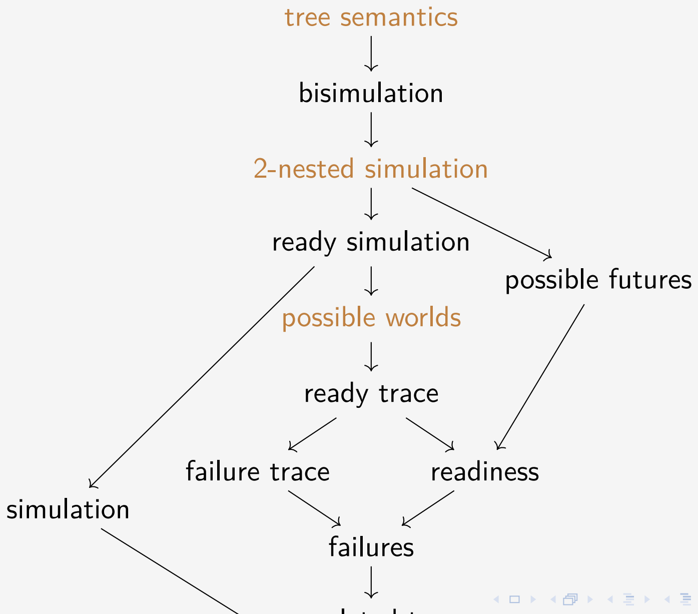
- $p \vDash A \bigwedge_{i\in V} \phi_i$ if $p \vDash A\phi_i$ for all $i \in V$, $A = I(p)$
- $p \vDash Aa\phi$ if $p \xrightarrow{a} q$, $q \vDash \phi$ and $A = I(p)$.

Two states are ready simulation equivalent if they satisfy the same (infinitary) ready simulation formulas.

$$M_n = F^n, \quad \eta = \mathrm{id}, \quad \mu^{nk} = \mathrm{id}$$

Ready Simulation: $F = \mathcal{P}\mathcal{P}(\Sigma \times \mathrm{Id}) \times \mathcal{P}\Sigma$, $\alpha(A) = (\mathcal{P}(A), \pi_1[A])$.

Linear Time – Branching Time Spectrum [van Glabeek]



Future Work

- Possible worlds semantics: some form of determinization requiring either multiple modified copies of LTS or something like $\mathcal{P}(\Sigma \times \mathsf{Id}) \to \mathcal{P}(\mathsf{Id}^{\Sigma})$ involved in a potential α.
- 2-nested simulation semantics: simulation formulas with limited negation
- tree semantics (graph isomorphism)
- probabilistic/weighted traces