Announcements and Updates in Coalgebraic Logic

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Introduction

- Dynamic epistemic logic captures evolving knowledge
- Coalgebraic logic unifies non-relational modal logics, e.g.
  - probabilistic/weighted/graded
  - game-theoretic (alternating-time)
  - conditional/counterfactual
  - neighbourhood-based
- Here: marry the two to cover
  - Probabilistic conditioning
  - Non-deterministic announcements
  - Announcements in weak epistemic logic

  etc. (Carreiro/Gorín/S. ICALP 2013)
- Tractability via local model updates.

Plan of the Talk

- Epistemic logic and public announcements
- Coalgebraic logic
- Coalgebraic announcement logic

Epistemic Logic

- Modalities K_i 'agent i knows'
- Kripke frames $(X, (R_i))$ (standardly: $S5_{,n}$)
- xR_iy: i considers y a possible alternative to x
- $E\phi = \bigwedge K_i\phi$ 'everybody knows ϕ'
- $C\phi = \nu X.(\phi \wedge EX)$ 'ϕ is common knowledge'

Public Announcements

- Dynamic epistemic logic: model evolves during formula evaluation
- Captures evolving (typically: increasing) knowledge
- Public announcement $[\psi]\phi$: ϕ holds when ψ is truthfully announced

$$M, x \models [\psi]\phi \iff (M, x \models \psi \implies \underbrace{M|_\psi}_{\text{updated model}}, x \models \phi)$$

where

$$M_\psi = (X \cap [\![\psi]\!], R \cap ([\![\psi]\!] \times [\![\psi]\!])).$$

- E.g. muddy children:

$$[\exists i.\, \mathrm{muddy}(i)]\, C((\forall j \neq i.\, K_i \neg \mathrm{muddy}_j) \to K_i\, \mathrm{muddy}_i).$$

Probabilistic Announcements

- Uncertain knowledge:
  - Distribution over epistemic alternatives
  - Operators $K_{i,p}$ 'agent i believes with probability $\geq p$ that'
  - Models = Markov chains $(X, (P_x)_{x \in X})$
- Update = conditioning: for $P_x(\psi) > 0$

$$(X, (P_x)_{x \in X})|_\psi = (X \cap [\![\psi]\!], (P_x(\_ \mid \psi)))$$

- E.g. myopic muddy children:

$$[\exists i.\, \mathrm{muddy}_i] C_1 (K_{i,1/2}(\forall j.\, \neg \mathrm{muddy}_j) \to K_{i,1/2} \mathrm{muddy}_i)$$

Coalgebraic Logic

- General modal similarity types (collections of modal operators)
- Abstract over the type of systems:
  - Fix $T : \mathbf{Set} \to \mathbf{Set}$
  - Systems = T-coalgebras

$$\xi : X \to TX$$

- Abstract over the interpretation of modal operators $\heartsuit$:
  - Predicate liftings $[\![\heartsuit]\!]_X : 2^X \to 2^{TX}$, natural in X
  - $x \models \heartsuit\phi$ iff

$$\xi(x) \in [\![\heartsuit]\!]_X([\![\phi]\!])$$

Example: Uncertain Knowledge

(Fagin/Halpern JACM 1994)

Functor $D(X)$ = distributions on X

Coalgebras $X \to D(X)^n$ = n-agent Markov chains

Operators $K_{i,p}$ 'agent i believes with probability $\geq p$'

$$[\![K_{i,p}]\!]_X(A) = \{\mu \in D(X) \mid \mu(A) \geq p\}$$

Example: Slow-Witted Agents

(Vardi LICS 1989)

Functor: $NX = 2^{(2^X)}$ (neighbourhoods), $TX = (NX)^n$.

T-coalgebras = n-agent neighbourhood models

$$[\![K_i]\!]_X(A) = \{f \in TX \mid [\![A]\!] \in f(i)\}$$

Agents cannot weaken or combine items of knowledge

(Variants, e.g. monotonicity)

Coalgebraic Logic: More Examples

- Alternating-time / coalitions
- Conditional / counterfactual logics
- Graded / weighted logics
- Modular combination (S./Pattinson ICALP 2007)

Coalgebraic Logic: Previous Results

- Finite model property for next-step logic (S. FOSSACS 2006)
- PSPACE upper bound for next-step logic (S./Pattinson LICS 2006)
- PSPACE tableaux (Pattinson/S. I&C 2010)
- PSPACE / EXPTIME for coalgebraic hybrid logic / coalgebraic DL with nominals
  (Myers/Pattinson/S. FOSSACS 2009, S./Pattinson/Kupke IJCAI 2009)
- EXPTIME for coalgebraic μ-calculus (Cirstea/Pattinson/Kupke CSL 2009)
- Completeness of flat coalgebraic fixed-point logics
  (S./Venema CONCUR 2011)
- NEXPTIME/EXPSPACE for coalgebraic fuzzy DL
  (S./Pattinson IJCAI 2011)
- Global Caching
  (Goré/Kupke/Pattinson TACAS 2010, Goré/Kupke/Pattinson/S. IJCAR 2010)

Deterministic Announcements

- Can replace update of carrier by update of transitions:

$$(X, \xi : X \to \mathcal{P}(X))|_\psi = (X, \lambda x.\, \xi(x) \cap [\![\psi]\!])$$

- Update = natural transform

$$\tau_X : \underbrace{TX \to (2^X \to TX)}_{\textit{local}};$$

  then

$$(X, \xi)|_\psi = (X, \tau(\_)([\![\psi]\!]) \circ \xi)$$

- τ strong announcement on Λ if

  $\tau_X(t)(A) \in TA$ and $t \models \heartsuit C \iff \tau_X(t)(A) \models \heartsuit C$ for $C \subseteq A, \heartsuit \in \Lambda$

- E.g. classical PAL, strong on $\{\Diamond\}$: $\tau_X(S)(A) = S \cap A$

Updates: Examples

- Graded modal logic / Bag functor:

$$\tau_X(\mu : X \to \mathbb{N} \cup \{\infty\})(A) = \mu|_A$$

  strong on $\{\Diamond_k \mid k \geq 0\}$

- Slow agents / Neighbourhoods:

$$\tau_X(f : n \to 2^{(2^X)})(A) = \lambda i.\,(f(i) \cap 2^A)$$

  strong on $\{K_i \mid i = 1, \ldots, n\}$

- Conditioning:

$$\tau_X(P)(A) = \begin{cases} P(\_ \mid A) & P(A) > 0 \\ P & \text{otherwise} \end{cases}$$

  update but not a strong announcement

Translating Strong Announcements

For $[\![\Delta]\!]$ update:

$$M, x \models \Delta_\psi \phi \iff M|_\psi, x \models \phi$$

- $\Delta_\psi \heartsuit \phi \equiv \heartsuit(\psi \wedge \Delta_\psi \phi)$ for $[\![\Delta]\!]$ strong on $\heartsuit$
- Hence: Strong announcements are invariant under behavioural equivalence.
- Upper bound for TBox reasoning inherited from base logic (EXPTIME)
  - avoid blowup by abbreviation via TBox
- Upper bound PSPACE for reasoning without TBox inherited from base logic (NP/PSPACE), given master modality $\boxdot$

$$\boxdot \top \qquad \boxdot \phi \to (\heartsuit \psi \leftrightarrow \heartsuit(\phi \wedge \psi))$$

Moving On

- Updates with effects: $\tau_X : TX \to (2^X \to FTX)$ plus predicate lifting λ for F, e.g.
  - Lossy announcements:
    $T = F = \mathcal{P}$, $\tau_X(S)(A) = \{S, S \cap A\}$,
    $\lambda = [\![\Box]\!]$ (demonic) or $\lambda = [\![\Diamond]\!]$ (angelic)
  - Probabilistically failing announcements
  - Unstable Markov chains
- Most general notion: regenerator = natural transform

$$\rho_X : 2^X \times 2^{TX} \to 2^{TX}$$

- Semantics over models and *global regenerators* $2^{TX} \to 2^{TX}$, initially *id*

Translating Regenerators

- (Regenerator ρ) $\circ$ (n-ary predicate lifting λ) = $n+1$-ary predicate lifting

$$(A, B_1, \ldots, B_n) \mapsto \rho_X(A, \lambda_X(B_1, \ldots, B_n))$$

  - Invariance under behavioural equivalence
  - Small model property
  - Under closure of $\mathrm{Prop}(\Lambda(V))$ under this operation:
    Same translation/complexity results as for updates

Example: Conditioning

$\Lambda = \{L_p \mid p \in [0,1] \cap \mathbb{N}\}$ not closed:

$$\Delta_\psi L_p \phi \equiv (\ell(\psi) = 0 \to \ell(\Delta_\psi \phi) \geq p) \wedge \ell(\psi \wedge \Delta_\psi \phi) \geq p \cdot \ell(\psi)$$

Solution: close Λ, e.g. admit linear inequalities

Result:

- Uncertain knowledge (w/o C) plus announcements is in PSPACE
- TBox reasoning in EXPTIME

Updates and Fixed Points

. . . can't be so hard:

$$\Delta_\psi(\nu X.\,\phi) = \nu X.\,\Delta_\psi\phi$$
$$\Delta_\psi X = X$$

Thus cover also variants of common knowledge

$$C\phi = \nu X.\,\phi \wedge EX$$

e.g.

$$C_p\phi = \nu X.E_p(\phi \wedge X)$$

('ϕ is commonly believed with confidence $\geq p$')

Conclusions

- Everything is coalgebraic ;)
- Can generalize announcements to coalgebraic setting
  - e.g. announcements to uncertain or thick agents
  - e.g. fallible announcements
- Invariance under behavioural equivalence
- Complexity bounds via translations
- New (?) definition of master modality

Westward Ho!

- Announcements plus fixed points / common knowledge
- Direct algorithms (tableaux)

That's it.

Thanks for your attention!