Coalgebraic announcement logics

Daniel Gorín
joint work with Facundo Carreiro and Lutz Schröder

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Executive summary

We generalize the public announcement operator from dynamic epistemic logics to the framework of coalgebraic modal logics. We give basic expressivity and complexity results.

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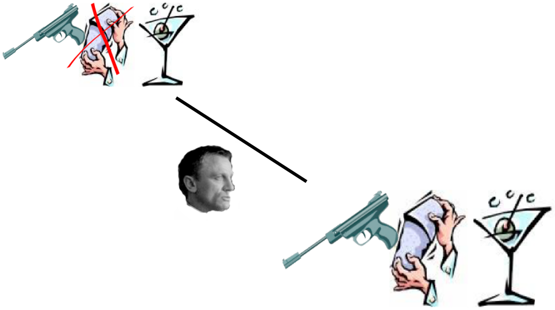
- Modal logics of *knowledge* and *beliefs*
- $K_\alpha \mathsf{light\_is\_off} \wedge \neg K_\alpha K_\beta\, \alpha\mathsf{\_is\_awake}$

- Modal logics of *knowledge* and *beliefs*
- $K_\alpha \mathsf{light\_is\_off} \wedge \neg K_\alpha K_\beta\, \alpha\mathsf{\_is\_awake}$
- Valid epistemic inferences:

Knowledge:	$K_\alpha \phi \rightarrow \phi$
Introspection (I):	$K_\alpha \phi \rightarrow K_\alpha K_\alpha \phi$
Introspection (II):	$\neg K_\alpha \phi \rightarrow K_\alpha \neg K_\alpha \phi$
Reasoning:	$K_\alpha(\phi \rightarrow \psi) \rightarrow K_\alpha \phi \rightarrow K_\alpha \psi$
Implicit:	$K_\alpha \phi$, if ϕ is a tautology

Epistemic logics – Possible world semantics



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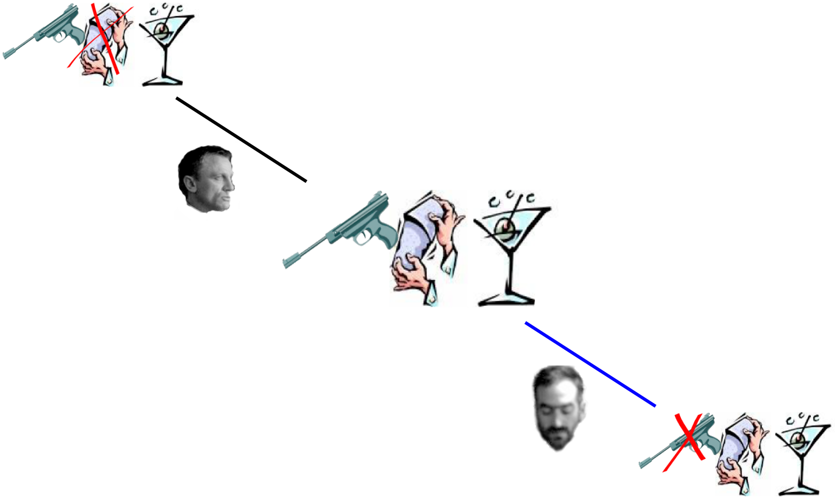

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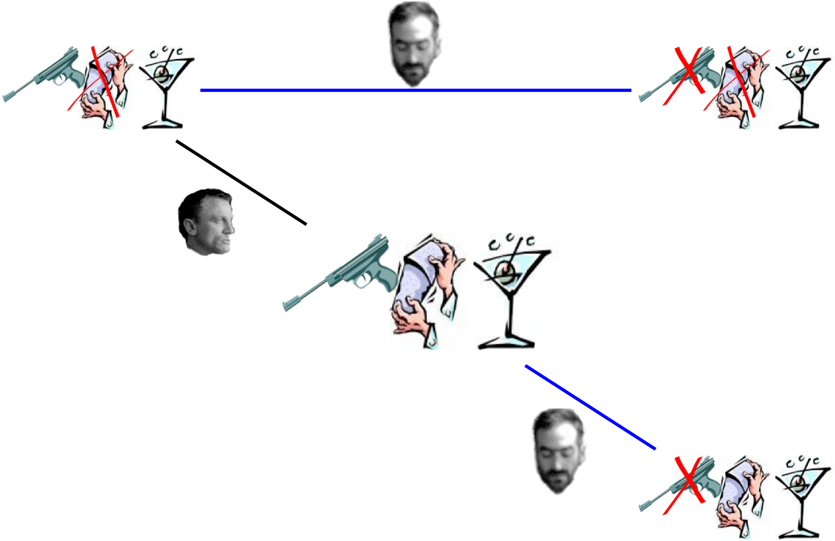

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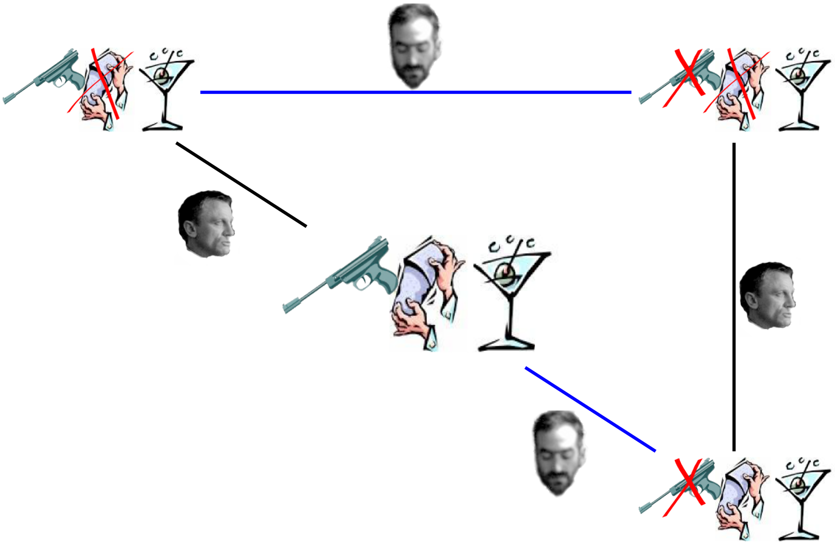
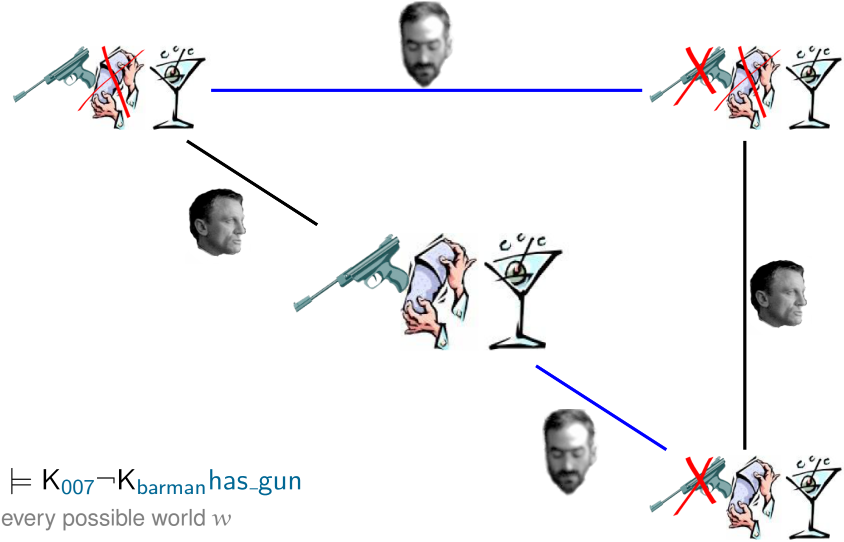


Epistemic logics – Possible world semantics

- Epistemic models are Kripke models $\mathcal{A} = \langle W, \{\sim_\alpha\}_{\alpha \in \mathsf{Ag}}, V \rangle$
  - $W \neq \emptyset$ (the set of *possible worlds*)
  - $\sim_\alpha \subseteq W \times W$ (equivalently, $\sim_\alpha : W \to \mathcal{P}W$)
  - $V : W \to \mathcal{P}(\mathsf{Prop})$
- where each $\sim_\alpha$ is an **equivalence relation** (usually)
- Satisfaction is just relational modal semantics, with K_α a “box”

$$\mathcal{A}, w \models \mathrm{K}_\alpha \phi \text{ iff } \mathcal{A}, w' \models \phi \text{ every time } w \sim_\alpha w'$$



$w \models K_{007} \neg K_{barman}\, has\_gun$

in every possible world w

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Dynamic epistemic logics

- Logics of knowledge and change
- Incorporate actions with epistemic impact

- **van Ditmarsch, van der Hoek and Kooi.** *Dynamic Epistemic Logic.* Springer, 2006.

Knowledge and epistemic change: the muddy children puzzle

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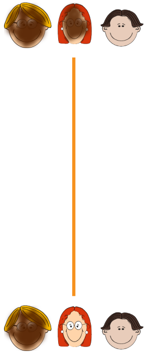

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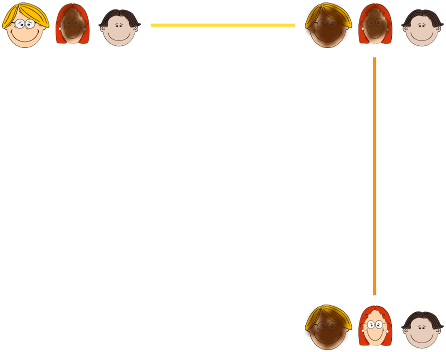

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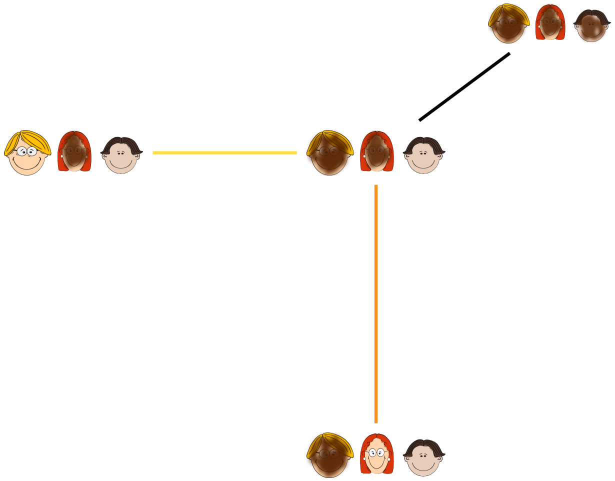

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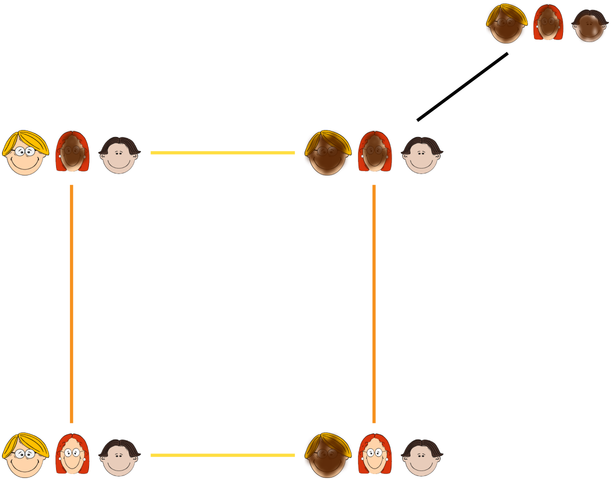

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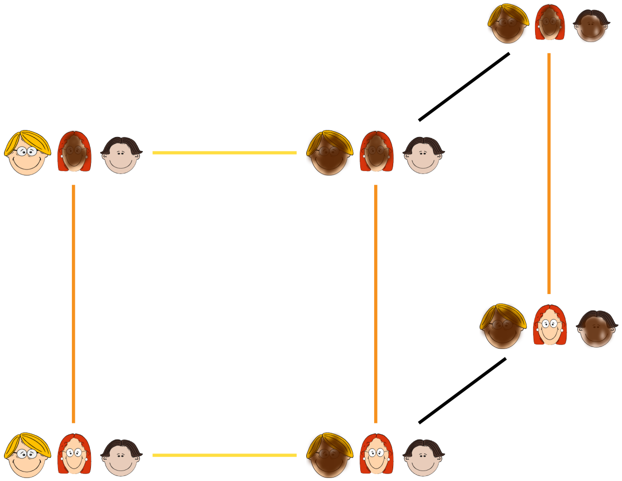

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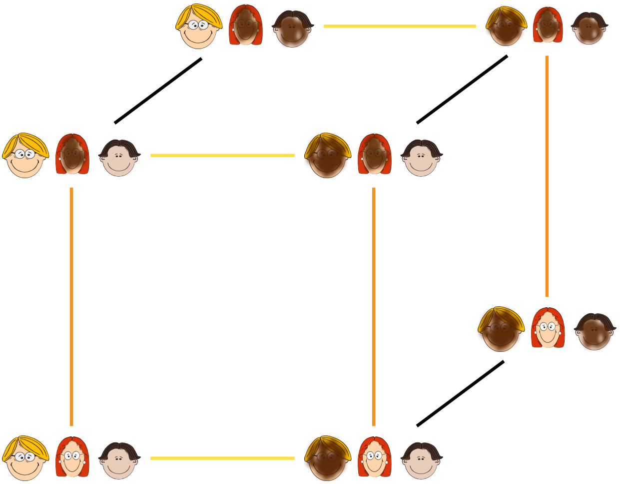

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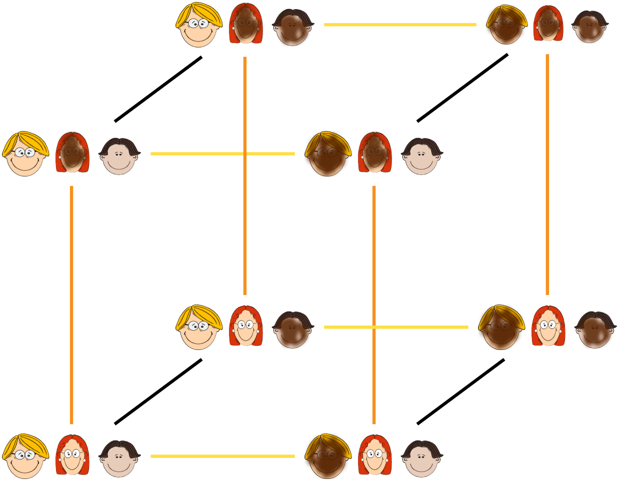

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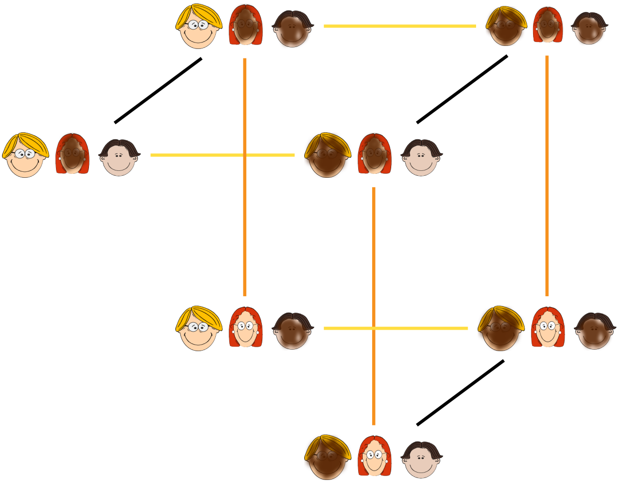

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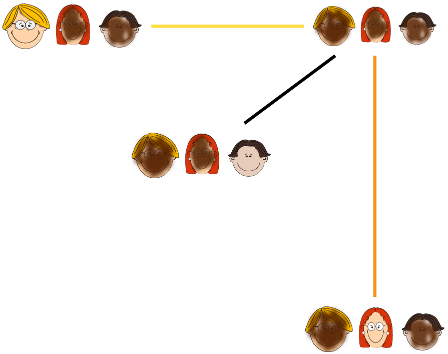

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The public announcement operator

- $[\phi]\psi \rightsquigarrow$ "after publicly (and faithfully) announcing ϕ, ψ holds"
- For example:

$$[\neg K_{\bullet}\mathsf{muddy} \wedge \neg K_{\bullet}\mathsf{muddy} \wedge \neg K_{\bullet}\mathsf{muddy}](K_{\bullet}\mathsf{muddy} \wedge \neg K_{\bullet}\mathsf{muddy})$$

- Semantics:

$$\mathcal{A}, w \models [\phi]\psi \text{ iff } \mathcal{A}|_{\phi}, w \models \psi, \text{ whenever } \mathcal{A}, w \models \phi$$

where $\mathcal{A}|_{\phi}$ is the restriction of $\mathcal{A}$ to the worlds that satisfy ϕ

- **Plaza.** *Logics of public communication.* ISMIS'89.

The public announcement operator

- This is a logic operator that *modifies* the models
- It is well-defined for arbitrary Kripke models

Some properties of the Public Announcement Logic (PAL)

- PAL is **not** more expressive than the base logic
  - removing nodes $\leftrightsquigarrow$ disconnecting nodes
  - rewrite rules:

$$[\phi]p \rightsquigarrow (\phi \rightarrow p) \qquad [\phi]K_\alpha \rightsquigarrow (\phi \rightarrow K_\alpha[\phi]\psi) \qquad \ldots$$

- But it is exponentially more succinct

  (both on epistemic and arbitrary models)

- While still in the same complexity class for satisfiability:
  - NP-complete in the (epistemic) single-agent case
  - PSPACE-complete for multi-agents (or arbitrary models)

- **Lutz.** *Complexity and succinctness of public announcement logic.* AAMAS'06.
- **French, van der Hoek, Iliev and Kooi.** *Succinctness of Epistemic Languages.* IJCAI'11.

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Announcements in a logic of probabilistic beliefs

- For an agent, some possible worlds are more likely true
- Probabilistic epistemic models: $\mathcal{A} = \langle W, \{\mu_\alpha\}_{\alpha \in \mathsf{Ag}}, V \rangle$
  - $\mu_\alpha : W \to D_\omega(W)$ (subject to frame conditions)
- $B_{\alpha,p}\phi \rightsquigarrow$ "agent α assigns to ϕ a likelihood of at least p"

$$\mathcal{A}, w \models B_{\alpha,p}\phi \text{ iff } \mu_\alpha(w)(A) = \sum_{\mathcal{A}, w' \models \phi} \mu_\alpha(w)(w') \geqslant p$$

Announcements in a logic of probabilistic beliefs

- Announcing (truthfully) a formula amounts to **conditioning**

$$\mathcal{A}, w \models [\phi]\psi \text{ iff } \mathcal{A}|_\phi, w \models \psi, \quad \text{whenever } \mathcal{A}, w \models \psi$$

where $\mathcal{A}|_\phi = \langle W, \{\tilde{\mu}_\alpha\}_{\alpha \in \mathsf{Ag}}, V \rangle$ with

$$\tilde{\mu}_\alpha(w) = \begin{cases} \lambda w'.\mu_\alpha(w)(w' \mid [\![\phi]\!]) & \text{if } \mu_\alpha(w)([\![\phi]\!]) > 0 \\ \mu_\alpha(w) & \text{otherwise} \end{cases}$$

$$[\![\phi]\!] = \{w \mid \mathcal{A}, w \models \phi\}$$

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- T is an endofunctor on Set
- A T-coalgebra is a tuple $\langle X, \gamma \rangle$ where $\gamma : X \to TX$
- The epistemic models are examples of coalgebras:

  Bond: Take $T := \mathcal{P} \times \mathcal{P} \times C_{\{\mathsf{gun},\mathsf{martini},\mathsf{was\_shaken}\}}$

  Children: Take $T := \mathcal{P} \times \mathcal{P} \times \mathcal{P} \times C_{\{\mathsf{muddy},\mathsf{muddy},\mathsf{muddy}\}}$

  Probabilistic: Take $T := \prod_{\alpha \in \mathsf{Ag}} D_\omega \times C_{\mathsf{Prop}}$
- Other examples: neighborhood models, various kinds of automata, transition systems…

- Λ is a set of modal operators
- Formulas: $\phi ::= \bot \mid \phi \to \phi \mid \heartsuit_k(\phi_1, \ldots \phi_k)$
- A k-ary modality $\heartsuit$ is interpreted using a predicate lifting $[\![\heartsuit]\!]$:
    - a natural transformation $[\![\heartsuit]\!] : \breve{\mathcal{P}}^k \dot{\to} \breve{\mathcal{P}}T$

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- The extension of ϕ in coalgebra $\langle X, \gamma \rangle$ is:

$$[\![\bot]\!]_\gamma := \emptyset$$

$$[\![\phi \to \psi]\!]_\gamma := \left(X \setminus [\![\phi]\!]_\gamma\right) \cup [\![\psi]\!]_\gamma$$

$$[\![\heartsuit(\phi_1 \ldots \phi_k)]\!]_\gamma := \left\{x \mid \gamma(x) \in [\![\heartsuit]\!]_X \left([\![\phi_1]\!]_\gamma \ldots [\![\phi_k]\!]_\gamma\right)\right\}$$

- For $T := \mathcal{P}$:

$$[\![\Diamond]\!]_X(A) := \{B \in \mathcal{P}X \mid B \cap A \neq \emptyset\}$$
$$[\![\Box]\!]_X(A) := \{B \in \mathcal{P}X \mid B \subseteq A\}$$

- For $T := D_\omega$ and for each $p \in [0;1] \cap \mathbb{Q}$:

$$[\![L_p]\!]_X(A) := \{\mu \in D_\omega X \mid \mu(A) \geqslant p\}$$
$$[\![M_p]\!]_X(A) := \{\mu \in D_\omega X \mid \mu(A) < p\}$$

- For $T := \mathcal{P} \times \mathcal{P} \times C_{\{\mathsf{gun},\mathsf{martini},\mathsf{was\_shaken}\}}$

$$[\![\mathsf{martini}]\!]_X := \{\langle R_{\mathsf{Bond}}, R_{\mathsf{Bartender}}, V\rangle \in TX \mid \mathsf{martini} \in V\}$$

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Wait! Why would you do such a thing?

- Epistemic models don't arise from a functor!

  (remember $\sim_\alpha$ is an equivalence)

- But model mutation is meaningful outside epistemic settings:
  - Resiliency checking (cf. sabotage logics)
  - Hypothetical querying and reasoning

- Public announcements are computationally well-behaved

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CML + dynamic modalities

- Π is a set of dynamic modal operators
- Formulas $\phi := \bot \mid \phi \to \phi \mid \heartsuit_k(\phi_1 \ldots \phi_k) \mid \Delta_\phi \phi$
- $\Delta_\phi \psi \rightsquigarrow$ "after announcing/assuming ϕ, ψ holds"
- How do we give meaning to each Δ?

Standard updates, revisited

- Announcing ϕ changes $\langle W, R, V \rangle$ to $\langle W, \tilde{R}, V \rangle$, where:

$$\tilde{R}(w) = \lambda w'.R(w)(w') \cap [\![\phi]\!]$$

- Announcing ϕ changes $\langle W, \mu, V \rangle$ to $\langle W, \tilde{\mu}, V \rangle$, where:

$$\tilde{\mu}(w) = \lambda w'.\mu(w)(w' \mid [\![\phi]\!])$$

- We'd like to interpret Δ_ϕ using a function $TX \to TX$
- Δ would be parametrized by a predicate $[\![\phi]\!]$: $\breve{\mathcal{P}}X \times TX \to TX$

- Formally, we interpret each $\Delta \in \Pi$ with an update $[\![\Delta]\!]$:
  - a natural transformation $[\![\Delta]\!] : T \dot{\to} (\breve{\mathcal{P}} \multimap T)$
  - where $\breve{\mathcal{P}} \multimap T$ is the Set-functor such that:

$$(\breve{\mathcal{P}} \multimap T)X := (TX)^{\breve{\mathcal{P}}X}$$

$$(\breve{\mathcal{P}} \multimap T)f := \lambda h \,.\, Tf \circ h \circ \breve{\mathcal{P}}f \qquad\qquad h : (TX)^{\breve{\mathcal{P}}X}$$

  - Naturality condition for $[\![\Delta]\!]$:

$$Tf\left([\![\Delta]\!]_X\left(t, \breve{\mathcal{P}}fA\right)\right) = [\![\Delta]\!]_Y\,(Tft, A)$$

here $f : X \to Y$, $t \in TX$ and $A \subseteq Y$.

Semantics of CML + updates

- Intuitively, we interpret Δ_ϕ applying $[\![\Delta]\!](-, [\![\phi]\!])$ everywhere:

$$[\![\bot]\!]_\gamma := \emptyset$$

$$[\![\phi \to \psi]\!]_\gamma := \Big(X \setminus [\![\phi]\!]_\gamma\Big) \cup [\![\psi]\!]_\gamma$$

$$[\![\heartsuit(\phi_1 \dots \phi_k)]\!]_\gamma := \Big\{x \mid \gamma(x) \in [\![\heartsuit]\!]_X \Big([\![\phi_1]\!]_\gamma \dots [\![\phi_k]\!]_\gamma\Big)\Big\}$$

$$[\![\Delta_\phi \psi]\!]_\gamma := [\![\psi]\!]_{[\![\Delta]\!]_X(-,[\![\phi]\!]_\gamma)\circ\gamma}$$

NB. $\langle X, [\![\Delta]\!]_X(-, [\![\phi]\!]_\gamma) \circ \gamma\rangle$ is a T-coalgebra!

Examples of updates

- For T $:= \breve{\mathcal{P}}$:

$$[\![\Delta]\!]_X(S, A) := S \cap A$$

- For T $:= D_\omega$:

$$[\![\Delta]\!]_X(\mu, A) := \begin{cases} \lambda x.\mu(x \mid A) & \text{if } \mu(A) > 0 \\ \mu & \text{otherwise} \end{cases}$$

- For T $:= \breve{\mathcal{P}}\breve{\mathcal{P}}$: (the **neighborhood** functor)

$$[\![\Delta]\!]_X(t, A) := t \cap \breve{\mathcal{P}}A$$

- For T $:= \mathcal{B}_\omega$ (the **bag** functor of graded modal logic)

$$[\![\Delta]\!]_X(b, A) := \lambda x.\mathtt{if}\ x \in A\ \mathtt{then}\ 0\ \mathtt{else}\ b(x)$$

Strong announcements

- One may expect more conditions from an "announcement":
  a. It disconnects all elements not satisfying the announcement

$$[\![\Delta]\!]_X(-, A) : TX \to TA$$

  b. The "essential" truth of the announcement doesn't change

$$t \in [\![\heartsuit]\!]_X(C) \text{ iff } [\![\Delta]\!]_X(t, A) \in [\![\heartsuit]\!]_X(C)$$

for all $t \in TX$, $C \subseteq A$, $\heartsuit \in \Lambda$

- A strong announcement on Λ is an update satisfying a and b

- NB. Condition b for Λ separating already guarantees naturality

Theorem

Let Λ consist of monotone operators. There is at most one strong announcement on Λ.

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Theorem

Let Δ be a strong announcement on Λ and let $\heartsuit \in \Lambda$. Then:

$$\Delta_\phi \heartsuit \psi \equiv \heartsuit \left(\psi \wedge \Delta_\phi \psi \right)$$

Corollary

If Π consists of strong announcements on Λ, then every formula ϕ is equivalent to an announcement-free formula ϕ^* over Λ.

Strong announcements – examples and counterexamples

- For T $:= \breve{\mathcal{P}}$, this is a strong announcement on $\{\diamond\}$ (not for $\{\Box\}$!):

$$[\![\Delta]\!]_X(S, A) := S \cap A$$

- For T $:= D_\omega$, this is not a strong announcement on any $\{L_p\}$:

$$[\![\Delta]\!]_X(\mu, A) := \begin{cases} \lambda x.\mu(x \mid A) & \text{if } \mu(A) > 0 \\ \mu & \text{otherwise} \end{cases}$$

- For T $:= \breve{\mathcal{P}}\breve{\mathcal{P}}$, this is a strong announcement on $\{\Box\}$:

$$[\![\Delta]\!]_X(t, A) := t \cap \breve{\mathcal{P}}A$$

- For T $:= \mathcal{B}_\omega$, this is a strong announcement on $\{\diamond_0, \diamond_1, \ldots\}$:

$$[\![\Delta]\!]_X(b, A) := \lambda x.\texttt{if } x \in A \texttt{ then } 0 \texttt{ else } b(x)$$

Announcements with *effects*

- The updates so far were *deterministic* in nature
- Consider instead a transformation $\mathrm{T} \dot{\to} (\check{\mathcal{P}} \rightharpoonup \mathcal{P}\mathrm{T})$:
  - $[\![\Delta]\!]_X(t, A)$ would give us a choice of transformations to t
  - Two readings for $\Delta_\phi \psi$:

    angelic: On *some* transformation induced by ϕ, ψ holds

    demonic: On *all* transformations induced by ϕ, ψ holds
- The type $\mathrm{T} \dot{\to} (\check{\mathcal{P}} \rightharpoonup \mathcal{P}\mathrm{T})$ is not enough to specify the behavior

  (but we can use predicate liftings!)

Announcements with *effects* – examples

- $F = \mathrm{Id}$, $\lambda = \mathrm{id}$ — the updates discussed earlier
- Non-deterministic updates: — $F = \breve{\mathcal{P}}$, $\lambda \in \{[\![\Diamond]\!], [\![\Box]\!]\}$
  - $T = \breve{\mathcal{P}}$, $\tau_X(S, A) := \{S \cap A, S\}$ — lossy announcements
  - $T = \breve{\mathcal{P}}$, $\tau_X(S, A) := \{S \setminus A, S\}$ — controlled sabotage
  - $T = \mathcal{S}_\omega$, $\tau_X^\varepsilon(\mu, A) = \{\tilde{\mu}_p \mid 0 \leqslant p \leqslant \varepsilon, \tilde{\mu}_p \in \mathcal{S}_\omega X\}$,
    where $\tilde{\mu}_p(x) := \texttt{if } x \in A \texttt{ then } \mu(x) + p \texttt{ else } \mu(x)$
    unstable (pseudo-)Markov chains
- Probabilistic updates — $F = \mathcal{D}_\omega$, $\lambda \in \{[\![L_p]\!] \mid p \in [0;1] \cap \mathbb{Q}\}$

Announcements with *effects* via *regenerators*

- We interpret Δ with a regenerator $[\![\Delta]\!] : \breve{\mathcal{P}} \times \breve{\mathcal{P}}T \dot{\to} \breve{\mathcal{P}}T$
- Given $\langle X, \gamma \rangle$ and a map $\rho : 2^{TX} \to 2^{TX}$ we define:

$$[\![\bot]\!]_{\rho,\gamma} := \emptyset$$

$$[\![\phi \to \psi]\!]_{\rho,\gamma} := \left(X \setminus [\![\phi]\!]_{\rho,\gamma}\right) \cup [\![\psi]\!]_{\rho,\gamma}$$

$$\left[\!\left[\Delta_\phi \psi\right]\!\right]_{\rho,\gamma} := [\![\psi]\!]_{[\![\Delta]\!]_X([\![\phi]\!]_{\rho,\gamma},-)\circ\rho,\gamma}$$

$$[\![\heartsuit\phi]\!]_{\rho,\gamma} := \left\{x \mid \gamma(x) \in \rho[\![\heartsuit]\!]_X[\![\phi]\!]_{\rho,\gamma}\right\}$$

- $[\![\phi]\!]_\gamma$ is then short for $[\![\phi]\!]_{\mathsf{id},\gamma}$

- $\tau : T \dot{\to} (\breve{\mathcal{P}} \to\!\!\triangleright FT)$ and $\lambda : (\breve{\mathcal{P}} \to \breve{\mathcal{P}}F)$ induce a regenerator ρ

$$\rho_X(A, S) := \breve{\mathcal{P}}(\tau_X(-)(A))\lambda_{TX}(S)$$

⚠ Non-deterministic announcement

$\neq$

non-deterministically picking a model

(except on tree-models)

(but the choice is always per state)

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Invariance under behavioral equivalence

- Let $\lambda'_X(A, B_1, \dots B_n) := \rho_X(A, \lambda_X(B_1, \dots B_n))$
- NB. λ' is a predicate lifting! (of higher arity)

Invariance under behavioral equivalence

- Let $\lambda'_X(A, B_1, \dots B_n) := \rho_X(A, \lambda_X(B_1, \dots B_n))$
- NB. λ' is a predicate lifting! (of higher arity)
- This gives a principle for eliminating "dynamic" modalities:

$$\Delta_\psi \heartsuit(\phi_1, \dots, \phi_n) \equiv \boxtimes_{(\Delta\cdot\heartsuit)}(\psi, \Delta_\psi\phi_1, \dots, \Delta_\psi\phi_n)$$

Theorem

Coalgebraic announcement logics are coalgebraic modal logics

Corollary

CALs are invariant under behavioral equivalence

Filtrations and the small (exponential) model property

- Coalgebraic modal logics have the exponential model property
- The (exponential) reduction of CAL to CML gives us a double-exponential model property
- But a filtration argument improves this result

Theorem

Every satisfiable formula of CAL has a model of exponential size

Corollary

Under very mild assumptions, the satisfiability problem (with global assumptions) for a CAL is in NEXPTIME

- Intuitively, we say that Λ is closed for Π if every $\boxtimes_{\Delta_1 \circ \heartsuit_1 \circ \cdots \circ \Delta_k \circ \heartsuit_k}(a_1, \ldots a_n)$ can be expressed with a Λ-formula of polynomial size (in n)
- E.g. when Π consists of strong announcements for Λ!

Theorem

If Λ is closed for Π, the satisfiability problem with global assumptions for $\mathsf{CAL}(\Pi, \Lambda)$ has the complexity of that for $\mathsf{CML}(\Lambda)$

Theorem

If Λ is closed for Π and has a mater modality, the satisfiability problem for $\mathsf{CAL}(\Pi, \Lambda)$ has the complexity of that for $\mathsf{CML}(\Lambda)$

($\boxdot$ is master if $\boxdot\top$ and $\boxdot\phi \to (\heartsuit\psi \leftrightarrow \heartsuit(\phi \wedge \psi))$ both hold)

Inherited complexited – examples

- We regain the known complexity for standard PAL
- Graded ML + strong announcements: PSPACE/EXPTIME
- (Monotone) Neighborhood logic + strong announcements: NP

- Non-example – probabilistic conditioning:
  - there is a master modality ✓
  - but announcements are not strong ×
  - we get optimum PSPACE complexity with an ad-hoc argument

Future work

- More examples!
- Generic succinctness results?
- Logics for hypothetical reasoning
  - Nominals to make them well-behaved?