LGruDat: Logical Foundations of Databases
Lecture 4 — FORC: finishing the basics

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A Decidability Detour

- Recursive enumerability of FORC
- R.e = recursive axiomatizability (Craig's trick, JSL 1953)
- Decidability of r.e. theories with finitely many Post-complete extensions
- What can we say about decidability of r.e. theories which put a finite upper bound on the size of their models?
- Is unrestricted FORC decidable then?

A variant of undecidability proof (adapted from Rob van Glabbeek)

- ...via Turing machines:
- one can use, e.g., undecidability of the language

$$\{(M, w) \mid M \text{ is the description of a Turing machine that accepts the string } w\}$$

- Assume the machine M in question has alphabet A and set of states Q
- think of individual elements as strings and put in Σ:
- ...a constant ϵ for empty string,
- ...a binary relation symbol S_a for every letter $a \in A$. $S_a(x, y)$ means $y = xa$
- ...a binary relation symbol R_q for every letter $q \in Q$. $R_q(x, y)$ means that M on input w can reach a configuration in state q with xy on the tape and the head of M pointing at the first position of y.
- The rest is routine

Another proof of undecidability

- via so-called Robinson Arithmetic Q
- (uses function symbols, though can be adopted)
- finitely axiomatized fragment of Peano Arithmetic
- (does not use induction scheme, which generates infinitely many formulas)
- Can you guess how the proof looks like?

(a wikipedia screenshot with axioms)

Robinson arithmetic (also called **Q**). Axioms (1) and (2) govern the distinguished element 0. (3) assures that S is an injection. Axioms (4) and (5) are the standard recursive definition of addition; (6) and (7) do the same for multiplication. Robinson arithmetic can be thought of as Peano arithmetic without induction. **Q** is a weak theory for which Gödel's incompleteness theorem holds. Axioms:

1. $\forall x \neg Sx = 0$
2. $\forall x \neg x = 0 \to \exists y \, Sy = x$
3. $\forall x \forall y \, Sx = Sy \to x = y$
4. $\forall x \, x + 0 = x$
5. $\forall x \forall y \, x + Sy = S(x + y)$
6. $\forall x \, x \times 0 = 0$
7. $\forall x \forall y \, x \times Sy = (x \times y) + x$.

What is going on in the finite?

- Is $\models^{\text{fin}}$ co-r.e.? Is $\models^{\text{fin}}$ r.e.?
- A brief glimpse at Trakhtenbrot's Theorem:
  satisfiability in the finite = halting problem

Reminder from yesterday

- How does the number of Post-complete extensions relate to length of maximal chain of extensions?
- How about finite axiomatizability of these extensions over the basic theory?
- Is $\models^{fin}$ compact?
- Is the class of finite models Δ-elementary?

Exact limits of first-order expressibility?

- We discussed the case of finiteness
- Try connectivity now: but what happens in the finite case?
- We need a more powerful/flexible tool, don't we?