LGruDat: Logical Foundations of Databases
Exercise 3, Deadline 21 Nov 2013

Tadeusz Litak

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1. Again, you **don't have to do the whole thing**. In fact, I would even ask you not to do all of it: given that this should not be new for you, it'd be a waste of paper, your time and my time. Just **pick any exercises that would sum up to 13 points**. You can pick single items from separate exercises, this is not a problem. This 13 points is 100 %. Don't worry, it is not a problem if you lose a point or two somewhere. Also, in every exercise you can treat all preceding ones as solved and use their results.
2. Finally, you can submit until **the beginning of the lecture (10:30 on Thu) either by email or in person**, whatever you prefer. In case we decide to meet as usual on Fri rather than Thu, the deadline is Friday.

1 Global versus local entailment

The corrected lecture notes were published online.

We defined

$$\Gamma \overset{\text{unr}}{\underset{loc}{\models}} \alpha \text{ iff } \mathsf{for\_all}\ \textbf{unrestricted}\ \mathfrak{A}, \kappa.(\mathfrak{A}, \kappa \models \Gamma \text{ implies } \mathfrak{A}, \kappa \models \alpha)$$

$$\Gamma \overset{\text{fin}}{\underset{loc}{\models}} \alpha \text{ iff } \mathsf{for\_all}\ \textbf{finite}\ \mathfrak{A}, \kappa.(\mathfrak{A}, \kappa \models \Gamma \text{ implies } \mathfrak{A}, \kappa \models \alpha)$$

$$\Gamma \overset{\text{unr}}{\underset{glo}{\models}} \alpha \text{ iff } \mathsf{for\_all}\ \textbf{unrestricted}\ \mathfrak{A}.$$

$$(\mathsf{for\_all}\ \kappa.\mathfrak{A}, \kappa \models \Gamma) \text{ implies } (\mathsf{for\_all}\ \kappa.\mathfrak{A}, \kappa \models \alpha)$$

$$\Gamma \overset{\text{fin}}{\underset{glo}{\models}} \alpha \text{ iff } \mathsf{for\_all}\ \textbf{finite}\ \mathfrak{A}.(\forall \kappa.\mathfrak{A}, \kappa \models \Gamma) \text{ implies } (\mathsf{for\_all}\ \kappa.\mathfrak{A}, \kappa \models \alpha)$$

Exercise **1.a** **(2 pts)** Is $\underset{loc}{\models} \subseteq \underset{glo}{\models}$, both in the finite and the infinite variant? Prove or give a counterexample

Exercise **1.b** **(2 pts)** How about the converse $\underset{glo}{\models} \subseteq \underset{loc}{\models}$? Prove or give a counterexample

Exercise **1.c** **(3 pts)** Define universal closure $\overline{\gamma}^{\forall}$ and $\overline{\Gamma}^{\forall}$. Use it to characterize the relationship between $\models_{loc}$ and $\models_{glo}$ (i.e., one of these relations holds between Γ and α iff another holds between $\overline{\Gamma}^{\forall}$ and α)

Exercise **1.d** **(3 pts)** In the lecture, we defined the syntactic relation $\vdash^{\text{unr}}_{loc}$. How one proves its soundness wrt $\models^{\text{unr}}_{loc}$? Don't give an entire detailed proof, but explain its structure and prove two of three clauses in detail. Is it also sound wrt $\models^{\text{unr}}_{glo}$? You can use one of above points to answer this question

Exercise **1.e** **(2 pts)** Can one modify $\vdash^{\text{unr}}_{loc}$ to get a relation $\vdash^{\text{unr}}_{glo}$ which has more chances to be complete wrt $\models^{\text{unr}}_{glo}$? I already suggested how during the lecture. Write a detailed definition (you don't have to quote all the axioms again though) and discuss how to update the proof of soundness in the previous point

2 Theories, completeness and Post-completeness

A theory is a set of **sentences** Γ closed under the syntactic deducibility relation $\vdash^{\text{unr}}_{loc}$, i.e., if $\Gamma \vdash^{\text{unr}}_{loc} \alpha$, then $\alpha \in \Gamma$

Exercise **2.a** **(1 pts)** Can we replace $\vdash^{\text{unr}}_{loc}$ with $\vdash^{\text{unr}}_{glo}$ from Exercise 1.e above? You can use all the results of that exercise in answering this question

Exercise **2.b** **(3 pts)** Use completeness to show that a set of sentences is a consistent Post-complete theory (a MCS) iff it is of the form $Th(\mathfrak{A})$ for some $\mathfrak{A}$

Exercise **2.c** **(4 pts)** Generalize: assume that Γ has no more than n Post-complete theories extending it (in the same signature of course). Show that there is no chain of proper extensions

$$T \subsetneq T_1 \subsetneq \cdots \subsetneq T_n$$

3 Elementarity and Δ-elementarity

Exercise **3.** **(5 pts)** Use compactness to show that the class of finite models is not EC_Δ (or not Δ-*elementary*, as I called it in the original version of lecture handouts)