# LGruDat: Logical Foundations of Databases
# Exercise 1, Deadline 8 Nov 2013

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I realize that the deadline is rather short. However

1. None of these exercises should be really new to you—this is a reminder of whatever basic logic course you were taking
2. More importantly, you **don't have to do the whole thing**! In fact, I would even ask you not to do all of it: given that this should not be new for you, it'd be a waste of paper, your time and my time. Just **pick any exercises that would sum up to 13 points**. You can pick single items from separate exercises, this is not a problem. This 13 points is 100 %. Don't worry, it is not a problem if you lose a point or two somewhere. Also, in every exercise you can treat all preceding ones as solved and use their results.
3. Finally, you can submit until **the beginning of the lecture (10:30 on Fri) either by email or in person**, whatever you prefer.

## 1 Exercises from the lecture

The corrected lecture notes were published online—-please note that they contain a number of changes to conventions and notation we are going to follow. Please familiarize yourself with the document, as well as the correction list.

Exercise **1.a** **(5 pts)** The slide ***Effects of queries***, page 3/4, contained a number of unsolved exercises marked by ⚠. The central one was a basic exercise from model theory: show that whenever

- $free(\alpha) = \{v_1, \ldots, v_n\}$
- $\kappa(v_1) = \kappa'(v_1), \ldots, \kappa(v_n) = \kappa'(v_n)$

then $\mathfrak{A}, \kappa \vDash \alpha$ iff $\mathfrak{A}, \kappa' \vDash \alpha$.

Exercise **1.b** **(1 pts)** Derive as a corollary another exercise from this slide: show equalities

$$\begin{aligned}
\phi(\mathfrak{A}) &:= \{(\mathsf{a}_1, \ldots \mathsf{a}_n) \in \mathsf{A}^n \mid \textsf{exists } \kappa.\kappa(\overline{\mathbf{v}}) = \overline{\mathsf{a}} \textsf{ and } \mathfrak{A}, \kappa \vDash \alpha\} \\
&= \{(\mathsf{a}_1, \ldots \mathsf{a}_n) \in \mathsf{A}^n \mid \textsf{for_all } \kappa.\kappa(\overline{\mathbf{v}}) = \overline{\mathsf{a}} \textsf{ implies } \mathfrak{A}, \kappa \vDash \alpha\} \\
&= \{(\mathsf{a}_1, \ldots \mathsf{a}_n) \in \mathsf{A}^n \mid \textsf{for_all } \kappa.\mathfrak{A}, \kappa[v_1 := \mathsf{a}_1] \ldots [v_n := \mathsf{a}_n] \vDash \alpha\} \\
&= \{(\mathsf{a}_1, \ldots \mathsf{a}_n) \in \mathsf{A}^n \mid \textsf{exists } \kappa.\mathfrak{A}, \kappa[v_1 := \mathsf{a}_1] \ldots [v_n := \mathsf{a}_n] \vDash \alpha\}
\end{aligned}$$

Exercise **1.c** **(1 pts)** Recall that according to one of corrections, we disposed of our *existential convention* allowing "underquantified" queries: that is, those for which $\overline{\mathbf{v}} \subsetneq free(\alpha)$. Choose an example of an equality in the previous exercise which would break down if underquantified queries are allowed and formally show its failure. That it, define a model and give an example of differing results of a single underquantified query.

Exercise **1.d** **(1 pts)** The last comment on the slide in question says *we obtain boolean queries as the limit case.* We call a query $\phi$ boolean iff for any $\mathfrak{A}$, $\phi(\mathfrak{A}) \in \{\mathsf{TRUE}, \mathsf{FALSE}\}$, where $\mathsf{TRUE} = \{\emptyset\}$ and $\mathsf{FALSE} = \emptyset$. What is the necessary and sufficient condition for a $\mathsf{FORC}$ query to be boolean?

## 2 Defining remaining connectives

Exercise **2.** **(4 pts)** Recall that

$$\begin{aligned} \alpha \to \beta &:= \neg(\alpha \wedge \neg\beta) \\ \alpha \vee \beta &:= \neg(\neg\alpha \wedge \neg\beta) \\ \alpha \leftrightarrow \beta &:= (\alpha \to \beta) \wedge (\beta \to \alpha) \\ \exists v.\alpha &:= \neg\forall v.\neg\alpha \end{aligned}$$

Formulate satisfaction condition for these connectives using metalinguistic implies , or , iff and exists and show that under the above definitions, these condition are indeed necessary and sufficient for satisfaction by $\mathfrak{A}, \kappa$.

## 3 Unrestricted validity

Write

$$\overset{\text{unr}}{\models} \alpha \text{ iff for\_all } \textbf{unrestricted } \mathfrak{A} \text{ and } \kappa.\ \mathfrak{A}, \kappa \models \alpha$$
$$\overset{\text{fin}}{\models} \alpha \text{ iff for\_all } \textbf{finite } \mathfrak{A} \text{ and } \kappa.\ \mathfrak{A}, \kappa \models \alpha$$

(of course, the tacit assumption is that the universal quantification is restricted to models adequate for $\Sigma$ where $\alpha \in \mathsf{FORC}(\Sigma)$). We will read these two symbols as *(finite) validity* and *unrestricted validity*—later on, we will also use them for *(finite) entailment* and *unrestricted entailment*. For the time being, let us focus on unrestricted validity: the notion which is the main subject of study of *classical model theory* (as opposed to *finite model theory*) and which was the main concern at your basic logic courses like GLoLoP.

Exercise **3.a** **(4 pts)**Which of the following schemes are valid and for which cases you can find a countermodel? In both cases provide an explicit argument. Remember that you can use the results of all previous exercises here:

$¿ \overset{\text{unr}}{\models}\ \forall v.(\alpha \wedge \beta) \leftrightarrow ((\forall v.\alpha) \wedge (\forall v.\beta))$ ?
$¿ \overset{\text{unr}}{\models}\ ((\forall v.\alpha) \vee (\forall v.\beta)) \rightarrow \forall v.(\alpha \vee \beta)$ ?
$¿ \overset{\text{unr}}{\models}\ (\forall v.(\alpha \vee \beta)) \rightarrow ((\forall v.\alpha) \vee (\forall v.\beta))$ ?
$¿ \overset{\text{unr}}{\models}\ (\forall v.(\alpha \vee \beta)) \rightarrow ((\forall v.\alpha) \vee \beta) \quad \Longleftarrow$ assuming $v$ fresh_for $\beta$?

Recall that a variable $v$ is fresh_for $\alpha$ if either does not occur in $\alpha$ at all or all its occurrences are within the scope of a quantifier.

Exercise **3.b** **(1 pts)** Different symbols for $\overset{\text{unr}}{\models}$ and $\overset{\text{fin}}{\models}$ already suggest they don't coincide, but it is natural to expect that at least implication in one direction would hold. Can you show that whenever we have $\overset{\text{unr}}{\models} \alpha$, we have also $\overset{\text{fin}}{\models} \alpha$?

# 4 A kind of Bonusaufgabe

Of course, with the scheme adopted for this Blatt, there is no point in using the work *Bonusaufgabe*, as strictly speaking no exercise is obligatory. But this one is demanding enough to merit such a name. Or maybe it isn't?

Exercise **4.** **(8 pts)** How about the converse of the previous exercise? Assume that $\Sigma$ contains a single binary symbol. Can you think of any $\alpha$ s.t. not $(\overset{\text{unr}}{\models} \alpha)$, but $\overset{\text{fin}}{\models} \alpha$? If so, give an explicit example of such a formula and prove both statements. As a bonus to Bonusaufgabe (i.e., you don't have to find the answer to get the full mark for this exercise), what happens if all symbols in $\Sigma$ are at most unary?